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Vibration analysis of mobile phone mast system by Rayleigh method

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ABSTRACT

To study the vibration of beams and columns, discretization techniques are required because such structures are continuous systems with infinite degrees of freedom. However, one can associate such systems to a system with a single degree of freedom, restricting the form to which the system will deform and describing their properties as a function of generalized coordinates. This technique is called the Rayleigh method. However, actual structures are more complex than simple beams and columns because their properties vary along their length. The objective of this work is to apply the technique recommended by Rayleigh to actual structures and find a single equation and correction factor that can be used to resolve practical problems in engineering. The structural elements selected for this study are metallic high-slenderness poles, for which the frequency of the first vibration mode were calculated analytically, as well as by finite element method-based computer modeling for comparative purposes. The results indicate that the analytical solution is 16% greater and 1% minor than the computational solution, and correction factors of 1.4 and 1.32 were found, respectively.

1. Introduction

The vibration response analysis of framed structures modeled as beams and columns has been studied by many researchers and continues to be treated extensively in the literature. Beams and columns constitute a continuous system with infinite degrees of freedom. To study the behavior of such systems, discretization techniques are required, wherein the structure is transformed into subsystems defined by points called joints. However, one can associate these subsystems to a system with a single degree of freedom, restricting the form to which the system will deform and describing their properties as a function of generalized coordinates. This technique was used by Rayleigh [1] in the study of the vibration of elastic systems, obtaining equations that were valid in the whole domain of the problem. However, actual structures are more complex than simple beams and columns because their properties vary along their length. In such cases, the Rayleigh method should be applied in parts and the integrals should be resolved within the limits established for each interval using the generalized properties calculated for each segment of the structure.

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It is important to note that axial compressive forces reduce the stiffness of the members of a structure. Geometric stiffness is a function of the normal force acting on a structural element and is due to the combined forces of the gravitational field and the self-weight of the structural element, as well as the devices attached to the element. Many well-known researchers in physics and mathematics have been drawn toward the investigation of the effect of the compression force on structural systems. In this sense, Timoshenko [2] pointed out that the first analytical model for understanding problems related to axially compressed bars could be attributed to Euler. More recently, Ratzersdorfer [3] presented a comprehensive study on the stability of compressed bars. Gambhir [4], who made important contributions to this field, stated that the studies of isolated bars are frequently related to the stability analysis of structural systems.

With regard to the experimental analysis of the dynamic behavior of slender columns, Brasil et al. [5] performed an experiment to determine the parameters of reduction in the stiffness of unstressed sections of an RC pole and correctly calculate the displacements of all sections. The experiment was performed for a reinforced concrete tower used for telecommunications, which had a length of 30 m and a circular ring cross section of 50-cm diameter.

The actual structures selected for this study were metallic high-slenderness poles, whose frequency of the first vibration mode was calculated by the Rayleigh method as described above, and one simplified equation was obtained. Computer models of these poles were developed using a geometric nonlinear solution by the finite element method (FEM) for comparative purposes. A relatively good agreement was obtained between the exact calculation and modeling results, with a difference of 16% and 1.1%, respectively for each case studied; hence, useful analytical solutions can be obtained independently, without the use of sophisticated computational programs, since a correction factor is applied to the simplified equation obtained from the analytical solution.

The rest of this paper is organized as follows. Initially, a review of the mathematical aspects of the problem is presented, followed by the determination of both the analytical and FEM solutions, although in many cases, both the solutions are already known. Then, the simulations for each mathematical formulation are presented. Finally, conclusions based on the principal results of this study are outlined.

2. Mathematical considerations based on Rayleigh method

The Rayleigh method [1], combined with the principle of virtual work, composes the mathematical basis of this study. Bert [6] stated that this modified method was originally applied by Rayleigh in 1894 to the one-dimensional problem of determining the fundamental frequency of a stretched string undergoing small-amplitude vibrations. The essence of the method is that it does not use a specific trial function with an undetermined coefficient for the deflection, as is done in the ordinary Rayleigh method.

The applications of the Rayleigh technique to mechanical systems with vibration problems are found in a wide range of scientific studies. Nikkhoo et al. [7] used the Rayleigh–Ritz method to obtain the natural frequencies and dynamic response of various beams under the excitation of a moving mass; in this method, trigonometric shape functions based on the end conditions of the beams were utilized. Moreover, they showed that a high level of accuracy could be obtained by utilizing a low number of shape functions, which had to be achieved to find the deformation field of various beams. Nguyen et al. [8] also used the Rayleigh–Ritz method to estimate the frequencies of poles and antenna masts while studying the aerodynamics of these structures. Along the same lines, Bhat [9] employed a method for calculating the natural frequencies of rectangular plates that have at least two parallel edges that are not simply supported and hence an exact solution cannot be obtained. Bhat inferred that the Rayleigh and Rayleigh–Ritz methods of analysis, when used with beam characteristic functions, could give good results in obtaining the natural frequencies of such plates. Chakraverty and Behera [10] investigated the free vibration of nonuniform Euler–Bernoulli nanobeams based on nonlocal elasticity theory. In this work, boundary characteristic orthogonal polynomials were implemented in the Rayleigh–Ritz method, which made the procedure computationally efficient because some elements of the mass and stiffness matrices of the generalized eigenvalue problem became either zero or one owing to the orthonormality of the assumed shape functions.

It is important to note that the technique developed by Rayleigh, and presented in his first book, was aimed to calculate the fundamental frequency of continuous elastic systems, which were transformed in systems with a unique degree of freedom. The precision of this method depends on the functional form used to represent the free vibration mode. The choice of an appropriate functional form is described by El Bikri et al. [11]; they performed a theoretical investigation of the geometrically nonlinear free vibrations of a clamped–clamped beam containing an open crack. Their approach involves the use of a semi-analytical model based on the extension of the Rayleigh–Ritz method to nonlinear vibrations; the model is mainly affected by the choice of the admissible functions.

Monterrubio and Ilanko [12] discussed the characteristics of sets of admissible functions to be used in the Rayleigh–Ritz method. Of particular interest were sets that could lead to converged results when penalty terms were added to the model constraints and elements were interconnected in vibration and buckling problems of beams as well as plates and shells of a rectangular planform. The discussion included the use of polynomials, trigonometric functions, and a combination of both. In the past, several sets of admissible functions have been used that have a limit on the number of terms that can be included in the solution without producing ill-conditioning. On the other hand, a combination of trigonometric and low-order polynomials have been found to produce accurate results without ill-conditioning for any number of terms and any number of penalty parameters that can be accommodated by the computer memory.

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