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A 8-neighbor model lattice Boltzmann method applied to mathematical–physical equations

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ABSTRACT

A lattice Boltzmann method (LBM) 8-neighbor model (9-bit model) is presented to solve mathematical–physical equations, such as, Laplace equation, Poisson equation, Wave equation and Burgers equation. The 9-bit model has been verified by several test cases. Numerical simulations, including 1D and 2D cases, of each problem are shown, respectively. Comparisons are made between numerical predictions and analytic solutions or available numerical results from previous researchers. It turned out that the 9-bit model is computationally effective and accurate for all different mathematical–physical equations studied. The main benefits of the new model proposed is that it is faster than the previous existing models and has a better accuracy.

1. Introduction

Lattice Boltzmann method (LBM) is a relatively new alternative of computational fluid mechanics. It was generated and developed from lattice gas automata (LGA) [1–3] and the kinetic theory of Boltzmann equation [4,5]. This method has been studied and researched for over 30 years since it was born, and it gradually became a hot topic worldwide. LBM is based on the mechanism of gas molecules. But, it is different from the traditional numerical methods. Besides, it is a discrete method in macroscopic scale, while, a continuous method in microscopic scale [6]. It is known that LBM can be employed in many research fields, such as microscopic flow [7], crystal growth [8], magnetic fluid [9,10], biological fluid [11,12], porous media flows [13–15], turbulence [16,17], burning chambers [18], multiphase flows [19,20], micro-nanoscopic and non-equilibrium flows [21,22], non-Newtonian and transcritical flows [23,24] etc., where the traditional numerical methods are very difficult to be applied. Many scholars have made great contributions in simulating mathematical–physical equations, such as, Laplace equation, Poisson equation, wave equation, Burgers equation, KdV equation, Schrödinger equation, Euler equation and N–S equation. The aim of this paper is to construct a series of 9-bit models as an inheritance and improvement of those predecessors' work [25–32]. Zhang et al., presented a 5-bit model in their work [28], this model works well in dealing with the Laplace equation. Chai and Shi presented a lattice Boltzmann model to solve the 2D and 3D Poisson equations [25], in the model they presented there was a genuine solver to the Poisson equation, the transient term was eliminated. For 2D Poisson equation, they developed a 5-bit model, which was tested by numerical cases. In 2000, Yan [27] developed a lattice Boltzmann model for 1D and 2D wave equations with truncation error of order two. In his paper, the author presented a 5-bit model and a 9-bit model with tested numerical cases. In his model, it is not necessary to have an ensemble average

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Nomenclature

c	the lattice sound speed
$C0_\alpha$	coefficients to be determined
$C1_\alpha$	coefficients to be determined
$C2_\alpha$	coefficients to be determined
$\vec{e}_\alpha$	unit velocities vector along discrete directions
$f(u)$	source term in mathematical–physical equations
F_α	out-force term of lattice Boltzmann equation
$F_\alpha^{(2)}$	multiple scale expansion term of out-force term of lattice Boltzmann equation
$f(\vec{r}, t)$	distribution functions
f_α	discrete distribution functions
$f_\alpha^{(1)}$	multiple scale expansion term of discrete distribution functions around f_α^{eq}
$f_\alpha^{(2)}$	multiple scale expansion term of discrete distribution functions around f_α^{eq}
f_α^{eq}	the equilibrium state of discrete distribution functions
f_α^{neq}	the non-equilibrium state of discrete distribution functions
$\vec{r}$	space position vector
$\vec{r}_b$	space position vector of point ***b***
$\vec{r}_f$	space position vector of point ***f***
$\vec{r}_{ff}$	space position vector of point ***ff***
$\vec{r}_w$	space position vector of point ***w***
Re	Reynolds number
t	time
t_1	expansion term of time scale
t_2	expansion term of time scale
t_0	present time step used in fourth order Runge–Kutta scheme
u	macroscopic quantities in mathematical–physical equations
u^{t_0}	u of present time step
$u^{t_0+\Delta t}$	u of next time step
$k_{1,\,2,\,3,\,4}$	parameters of fourth order Runge–Kutta scheme
α	discrete directions
β	a parameter of wave equation to be determined
Δ_e	embed depth
Δt	time step
Δx	grid spacing
ε	small Knudsen number
λ	a parameter to be determined
ν	kinematic viscosity coefficient
σ_{ij}	Kronecker symbol
τ	single relaxation time
ω_α	weight coefficient
$\bar{\omega}_\alpha$	weight coefficient in Chai's model
∇^2	Laplace operator
∇u	gradient of macroscopic quantity u
∇	partial differential operator
∇_1	space expansion term of partial differential operator

Superindices

αi	α represents discrete directions and $i=1$, 2 represents the coordinates in x and y directions
αj	α represents discrete directions and $j=1$, 2 represents the coordinates in x and y directions
eq	represents equilibrium
neq	represents no-equilibrium

to get the macroscopic quantity, so the statistical errors disappear. Duan and Liu [26] developed a special lattice Boltzmann model to simulate 2D unsteady Burgers equation. The maximum principle and the stability were proved in their work. Their study indicates that lattice Boltzmann model is highly stable and efficient even for the problems with severe gradient. This model is a 4-bit model without the stationary state in discrete velocities. They developed another lattice Boltzmann model to solve the modified Burgers equation in 2008 [30]. In this new paper, they presented a 2-bit model without stationary state in discrete velocities for 1D modified Burgers equation. Zhang and Yan [32] proposed a higher-order moment lattice

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