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The morphological stability of dendritic growth from the binary alloy melt with an external flow

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ABSTRACT

The morphological stability of dendritic growth from the binary alloy melt with an external flow is studied by means of the matched asymptotic expansion method and multiple variable expansion method. The uniformly valid asymptotic solution is obtained for the case of the large Schmidt number. The analytical result reveals that the stability of dendritic growth depends on a critical stability number above which dendritic growth is stable. The selection condition of dendritic growth determines the Peclet number, tip growth velocity, tip radius and oscillation frequency, which is significantly affected by the external flow. The stability mechanism of dendritic growth in the binary alloy melt with the external flow remains the same as that in pure melt. In the binary alloy melt with the external flow the solute concentration destabilizes the dendritic growth system. The numerical computation for various growth conditions demonstrates the variations of the critical stability number, tip growth velocity, tip radius, and oscillatory frequency with the undercooling, external flow and morphological number.

1. Introduction

The pattern formation of dendritic growth is one of the fundamental subjects in materials science and engineering. The morphological instability during solidification may lead to various dendritic microstructures and greatly affect the physical and mechanical properties of final solidification products. During the past decades, several theories on the morphological stability of dendritic growth have been developed successively: maximocity principle [1], marginal stability hypothesis [2], microscopic solvability condition theory [3] and interfacial wave theory [4]. The dendritic growth system is not only a nonlinear partial differential equation problem which contains the nonlinear Navier–Stokes equation, but also a singularly-perturbed moving boundary value problem in which the moving interface shape is also a part of solution. Generally, it is hard or even impossible to obtain an exact solution, but approximate analytical solutions are possible. Bouissou et al. [5] and Ananth et al. [6,7] studied the effect of convective flow on dendritic growth in the pure melt, but their analytical solutions were not uniformly valid. By using the matched asymptotic expansion method, Xu [8] studied dendritic growth from the pure melt with an external flow and obtained the uniformly valid asymptotic solution as well as the stability criterion of the interface morphology. Compared with the case in the pure melt, the dynamics of dendritic growth in a convective binary alloy melt may be greatly changed because of the effect of constitutional undercooling and melt convection [9–14]. Although we may assume that the dynamics of dendritic growth in the convective binary alloy melt obeys the same mechanism as that in the convective pure melt, we need perform strict mathematical argumentation to reveal this mechanism.

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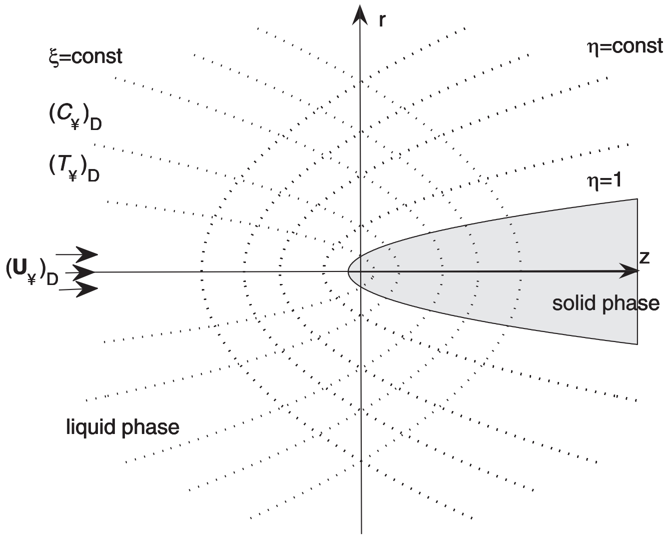

Fig. 1. The schematic diagram of dendritic growth in an undercooled alloy melt affected by an external flow.

Chen et al. [15] obtained the uniformly valid asymptotic solution for dendritic growth from the binary alloy melt with an external flow and showed the significant effect of melt convection on the interface shape, temperature field and concentration field. The aim of the paper is to carry out the morphological stability analysis of dendritic growth from the binary alloy melt with an external flow. For the stability analysis, there are two basic approaches: one is to solve an initial and boundary value problem, the other is to make a normal mode analysis in which the perturbations are assumed in the form of quasi-steady waves and we transform their evolution into an eigenvalue problem. In the present paper, we will adopt the normal mode approach developed by Xu [4] to investigate the uniformly valid asymptotic solution of perturbations by using the matched asymptotic expansion method and multiple variable expansion method, reveal the stability mechanism of dendritic growth and obtain the explicit formulae of the tip growth velocity, tip radius and oscillation frequency for dendritic growth in the binary alloy melt with an external flow.

2. Mathematical formation

We consider the interface of a single axi-symmetric dendrite growing in the convective undercooled binary alloy melt driven by an external flow (Fig. 1). It is assumed that the melt is the isotropic and incompressible Newtonian fluid. $(T_\infty)_D$ and $(C_\infty)_D$ denote the dimensional temperature and concentration far from the dendrite interface, respectively. $(T_L)_D$ and $(T_S)_D$ denote the dimensional temperature fields in the liquid phase and solid phase, respectively. $(C_L)_D$ and $(C_S)_D$ denote the dimensional concentration fields in the liquid phase and solid phase, respectively. $(\mathbf{U})_D$ denotes the dimensional velocity field in the liquid phase. Herein the subscripts "D" represent dimensional quantities. The nondimensionalization is carried out in the same way as that in Ref. [15], in which the constant tip growth velocity V is the velocity scale, the diffusion length ℓ_D is the length scale and ℓ_D/V is the time scale. In the moving paraboloidal coordinate system (ξ, η, θ) fixed at the dendrite tip, $r = \eta_0^2\xi\eta,\ z = \eta_0^2(\xi^2 - \eta^2)/2,$ where the normalization parameter η_0^2 is to be determined by setting the location of the dendrite-tip,$r = \sqrt{x^2 + y^2}$. Since there is no flow in the azimuthal direction θ, the interface shape function is written as $\eta = \eta_S(\xi, t)$, the velocity field is written as $\mathbf{U} = (u, v, 0)$, the vorticity vector is written as $\Omega = \nabla \times \mathbf{U} = (0, 0, \omega_3)$, where u and v denote the components along ξ direction, η direction, respectively, and ω_3 is the only non-zero third component of Ω. We introduce the stream function $\psi = \psi(\xi, \eta, t)$ and vorticity function $\zeta(\xi, \eta, t) = \eta_0^2\xi\eta\omega_3$ such that

$$u = \frac{1}{\eta_0^4\xi\eta\sqrt{\xi^2 + \eta^2}}\frac{\partial\psi}{\partial\eta}, \quad v = \frac{-1}{\eta_0^4\xi\eta\sqrt{\xi^2 + \eta^2}}\frac{\partial\psi}{\partial\xi}, \tag{2.1}$$

where η_0^2 is determined such that the steady interface function satisfies the normalization condition $\eta_S(0, t) = 1$. For the binary alloy melt the temperature T_S and concentration C_S in the solid phase are taken to be constant, the densities in the liquid phase and solid phase are taken to be equal and the buoyancy effect is neglected.

For the typical metals, the Schmidt number S_C is large, $S_C = \upsilon/\kappa_D$, where υ is the kinematical viscosity, κ_D is the solute diffusivity in the liquid phase. We adopt the reciprocal of the large Schmidt number $\varepsilon_2 = 1/S_C$ as a small parameter. By using the matched asymptotic expansion method we obtained the solution for the temperature, concentration, flow fields and the interface shape function [15]. The basic state solution is denoted by the subscript B.