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Frequency dependent iteration method for forced nonlinear oscillators

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ABSTRACT

A new iteration method for nonlinear vibrations has been developed by decomposing the periodic solution in two parts corresponding to low and high harmonics. For a nonlinear forced oscillator, the iteration schema is proposed with different formulations for these two parts. Then, the schema is deduced by using the harmonic balance technique. This method has proven to converge to the periodic solutions provided that a convergence condition is satisfied. The convergence is also demonstrated analytically for linear oscillators. Moreover, the new method has been applied to Duffing oscillators as an example. The numerical results show that each iteration schema converges in a domain of the excitation frequency and it can converge to different solutions of the nonlinear oscillator.

1. Introduction

The harmonic balance method (HBM) has many applications in nonlinear dynamics. The original principle of this method is to express the periodic solution in terms of Fourier series with limited numbers of harmonics and to substitute this expression to the dynamic equation in order to find out balance of all harmonics. A typical difficulty of this method is linked to the dependence of the quality of the approach on the way to carry enough terms in the solution and check the order of the coefficients for all the neglected harmonics [1,2].

Some alternative techniques based on HBM have been developed in order to solve this difficulty. The Galerkin procedure [3–6] was used to calculate incrementally the Fourier coefficients of the classical HBM. Another technique called the iteration procedures were presented by Mickens [7,8] and applied to different kinds of non-linearities [9,10]. The high dimensional harmonic balance (HDHB) was developed by Hall et al. [11]. This method is based on a constant Fourier transformation matrix which is an approximation of the matrix related to the non-linearity deduced from the classical HBM. This method has some advantages in calculation in case of high dimension, but it can lead to non-physical solutions [12–14]. Recently, other new methods based on HBM were developed by Cochelin et al. [15–17], Ju and Xue [18,19] and Ju [20].

In this paper, we propose a new method for forced nonlinear oscillators based on the iteration procedures and HBM. The existing methods propose to use the same iteration schema for all harmonics. This new method provides different schemas which adapt to the frequency of the force. By decomposing the periodical solution in low and high harmonic components, an iteration schema is presented and then developed by using HBM. The schemas are proved to converge to the periodic solution, provided that a sufficient condition is satisfied. Thus, the harmonic decomposition is a new way to build iteration schemas which could be applied to other problems using HMB (e.g., the high dimensional problems). Moreover, this method

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is demonstrated to converge to the analytic solution of a linear oscillator and it is applied to a forced Duffing oscillator. The numerical results show that each iteration schema converges in each range of the excitation frequency and it can converge to different solutions of the oscillator.

2. Iteration schemas

Let's consider a forced nonlinear oscillator given by

$$\frac{d^2u}{dt^2} + \beta\dot{u} + \omega_0^2 u + f(u,\dot{u}) = A\cos\Omega t, \tag{1}$$

where $f(u,\dot{u})$ is a nonlinear function. This equation is rewritten to read

$$\frac{d^2u}{dt^2} + \Omega^2 u = (\Omega^2 - \omega_0^2)u - \beta\dot{u} - f(u,\dot{u}) + A\cos\Omega t \equiv G(u,\dot{u}). \tag{2}$$

A classical iteration schema is often proposed as follows:

$$\frac{d^2u_{k+1}}{dt^2} + \Omega^2 u_{k+1} = \mathcal{L}(G(u_i,\dot{u}_j)_{i,j\le k}), \tag{3}$$

where $\mathcal{L}$ is a linear form which is chosen for each function $G(u,\dot{u})$ in order to meet the convergence. Now we will build a different iteration schema by considering the periodic solution defined by series $\{q_{nk}\}$, $\{p_{nk}\}$ (with $1 \le n \le N$) as follows:

$$u_{Nk}(t) = \frac{q_{0k}}{2} + \sum_{n=1}^{N} q_{nk}\cos n\Omega t + p_{nk}\sin n\Omega t. \tag{4}$$

For each $\mathcal{N}$ $(0 \le \mathcal{N} \le N)$, we decompose u_{Nk} into two terms which correspond to low and high harmonics as follows:

$$X_k(t) = \frac{q_{0k}}{2} + \sum_{n=1}^{\mathcal{N}} q_{nk}\cos n\Omega t + p_{nk}\sin n\Omega t, \tag{5}$$

$$Y_k(t) = \sum_{n=\mathcal{N}+1}^{N} q_{nk}\cos n\Omega t + p_{nk}\sin n\Omega t, \tag{6}$$

$$u_{Nk}(t) = X_k(t) + Y_k(t). \tag{7}$$

Then, the proposed iteration schema is

$$\frac{d^2Y_{k+1}}{dt^2} + \beta\dot{X}_{k+1} + \omega_0^2 X_{k+1} = A\cos\Omega t - f(u_{Nk},\dot{u}_{Nk}) - \frac{d^2X_k}{dt^2} - \beta\dot{Y}_k - \omega_0^2 Y_{k.} \tag{8}$$

Here, we take the initial values (i.e., $k = 1$) by $q_{n1} = p_{n1} = 0$. Now we will develop this schema by using the harmonic balance technique. By performing the Fourier series development of Eq. (8), we obtain the following results:

$$\begin{aligned}\frac{1}{T}\int_0^T \left(\frac{d^2Y_{k+1}}{dt^2} + \beta\dot{X}_{k+1} + \omega_0^2 X_{k+1}\right)\cos n\Omega t\ dt &= A\delta_{1n} - \mathcal{C}_{nk} \\ &\quad -\frac{1}{T}\int_0^T \left(\frac{d^2X_k}{dt^2} + \beta\dot{Y}_k + \omega_0^2 Y_k\right)\cos n\Omega t\ dt,\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{1}{T}\int_0^T \left(\frac{d^2Y_{k+1}}{dt^2} + \beta\dot{X}_{k+1} + \omega_0^2 X_{k+1}\right)\sin n\Omega t\ dt &= -\mathcal{S}_{nk} \\ &\quad -\frac{1}{T}\int_0^T \left(\frac{d^2X_k}{dt^2} + \beta\dot{Y}_k + \omega_0^2 Y_k\right)\sin n\Omega t\ dt,\end{aligned} \tag{10}$$

where $T = \frac{2\pi}{\Omega}$; $\delta_{1n} = 1$ if $n = 1$ and $\delta_{1n} = 0$ if other. $\mathcal{C}_{nk}, \mathcal{S}_{nk}$ are the Fourier coefficients of $f(u_{Nk},\dot{u}_{Nk})$,

$$\begin{cases} \mathcal{C}_{nk} = \dfrac{2}{T}\displaystyle\int_0^T f(u_{Nk},\dot{u}_{Nk})\cos n\Omega t dt \\ \mathcal{S}_{nk} = \dfrac{2}{T}\displaystyle\int_0^T f(u_{Nk},\dot{u}_{Nk})\sin n\Omega t dt. \end{cases} \tag{11}$$