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Novel dynamics of a predator–prey system with harvesting of the predator guided by its population

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ABSTRACT

A predator–prey model was extended to include nonlinear harvesting of the predator guided by its population, such that harvesting is only implemented if the predator population exceeds an economic threshold. The proposed model is a nonsmooth dynamic system with switches between the original predator-prey model (free subsystem) and a model with nonlinear harvesting (harvesting subsystem). We initially examine the dynamics of both the free and the harvesting subsystems, and then we investigate the dynamics of the switching system using theories of nonsmooth systems. Theoretical results showed that the harvesting subsystem undergoes multiple bifurcations, including saddle-node, supercritical Hopf, Bogdanov–Takens and homoclinic bifurcations. The switching system not only retains all of the complex dynamics of the harvesting system but also exhibits much richer dynamics such as a sliding equilibrium, sliding cycle, boundary node (saddle point) bifurcation, boundary saddle-node bifurcation and buckling bifurcation. Both theoretical and numerical results showed that, by implementing predator population guided harvesting, the predator and prey population could coexist in more scenarios than those in which the predator may go extinct for the continuous harvesting regime. They could either stabilize at an equilibrium or oscillate periodically depending on the value of the economic threshold and the initial value of the system.

1. Introduction

Mathematical models play a vital role in investigating the optimal management and exploitation of renewable resources. A number of researchers have examined the influence of harvesting strategies on the interaction of different species [1–4]. For example, predator-prey models with linear and constant harvesting regimes were considered by researchers [3–12]. The dynamics of models with a linear harvesting regime are similar to those with linear death rates. However, models with a constant harvesting regime could exhibit more complicated dynamics. Brauer and Soudack [4] discussed a ratio-dependent predator-prey model with constant predator harvesting and indicated the existence of a limit cycle and homoclinic bifurcation. Xiao and Ruan [13] further studied this model and by choosing a constant harvest rate and the death rate as bifurcation parameters, they found that the model could undergo multiple bifurcations including saddle-node, Hopf, homoclinic and Bogdanov–Takens bifurcations. Although linear and constant harvesting have been widely used, the most realistic harvest strategy is that when the harvested yield initially increases with population and saturates at a relatively

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high level when the population is sufficiently large [9]. Some ratio-dependent models with a saturated harvesting strategy have been studied [14–17]. However, much remains unknown about the effect of nonlinear or saturated harvesting on the dynamics of the traditional predator-prey model. So, one of the purposes of this study is to examine whether nonlinear predator harvesting can induce more complicated dynamics in a traditional predator-prey model.

Continuous harvesting models have been widely studied but their assumptions are questionable since regardless of the population of the predator and prey, they may lead to the extinction of the predator or the prey. So, a state dependent harvesting strategy has been proposed and modeled [18,19]. On the basis of impulsive difference equations, this proposed modeling approach assumes that the control measure is implemented when the prey or predator population reaches an economic threshold. However, it failed to describe persistent implementation of harvest measures. So, another purpose of this study is to propose a model which can describe a harvesting policy guided by the predator or prey population and, also, this policy is implemented persistently.

Recently, nonsmooth dynamic systems have been used to study threshold policies in various applied fields [20–26] such as plant disease control and human infectious disease control. Moreover, nonsmooth dynamic systems have also been used recently to study a harvesting policy with a threshold regime, in which harvesting is not implemented instantaneously but piecewise continuously [27]. However, in that paper, only linear harvesting was considered and bifurcations caused by a discontinuous harvesting policy were not studied. Based on this threshold policy we extend the predator-prey model by including a predator population guided harvesting strategy. In particular, a saturated harvesting strategy is implemented only if the predator population exceeds a critical threshold. Biologically, the proposed model addresses a realistic harvesting policy under which sustainable development of two species is achieved, i.e., we focus on the effects of nonlinear harvesting and a threshold policy on the coexistence of the predator and the prey. In addition, we also theoretically concentrate on what kinds of novel dynamics and bifurcations will occur when the traditional model is perturbed by a nonlinear harvesting regime and when the threshold policy is implemented.

This paper is organized as follows. In the second section, the model is proposed and the dynamical behaviors of the two subsystems are studied. In the third section, the dynamics of the switching system including existences and stabilities of equilibria and limit cycles and existences of bifurcations are discussed. Also, some numerical simulations are done to verify the major analytical findings in the second and third sections. Finally the biological conclusions on effect of saturated and predator population guided harvesting are addressed.

2. Models and preliminaries

Our study aims to establish a predator-prey model with switching between a traditional model (the free system) and a model with a nonlinear harvesting regime for the predator population (the harvesting system). The free system is shown as follows:

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{K}) - \beta xy \\ \frac{dy}{dt} = k\beta xy - dy \end{cases} \tag{1}$$

where x and y denote the numbers of prey (small fish) and predators (big fish) at time t, respectively. r is the growth rate of the prey, K is the carrying capacity of the prey, β is the per-capita rate of predation of the predator, k is a constant conversion rate of eaten prey into new predator abundance and d is the death rate of the predator. Following the ideas of Lenzini and Rebaza [15], the harvesting system is established as follows:

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{K}) - \beta xy \\ \frac{dy}{dt} = k\beta xy - dy - \frac{qy}{1+\omega y} \end{cases} \tag{2}$$

The term $\frac{qy}{1+\omega y}$, which is a saturation function, represents the harvesting of the predator. ω is a suitable constant and q is the rate of harvesting. All the parameters are positive constants.

Bearing the need for sustainable development in mind, we employ an economic threshold (denoted by P) of the predator to determine whether or not the harvesting strategy is implemented. When the number of predators is less than P the free system functions, but when the number of predators becomes greater than P nonlinear harvesting of the predator is implemented. Our switching system reads:

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{K}) - \beta xy \\ \frac{dy}{dt} = k\beta xy - dy - \varepsilon\frac{qy}{1+\omega y} \end{cases} \tag{3}$$

with

$$\varepsilon = \begin{cases} 0, & y < P \\ 1, & y > P \end{cases} \tag{4}$$

The region below the switching surface (namely, $y < P$) is denoted by (S_1) and that above the economic surface (namely, $y > P$) is denoted by (S_2).