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Free vibration of four-parameter functionally graded moderately thick doubly-curved panels and shells of revolution with general boundary conditions

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ABSTRACT

This paper aims to present a unified vibration analysis approach for the four-parameter functionally graded moderately thick doubly-curved shells and panels of revolution with general boundary conditions. The first-order shear deformation theory is used in this formulation. The functionally graded panels structures consists of ceramic and metal which are set to vary continuously in the thickness direction according to the general four-parameter power-law distribution, and six types of power-law distributions are considered for the ceramic volume fraction. The admissible function of the FG panels and shells of revolution is obtained by the improved Fourier series with the help of the governing equations and the boundary conditions. The solution is obtained by using the variational operation in terms of the unknown expanded coefficients. By a great many numerical examples, the rapid convergence and good reliability and accuracy of the proposed approach are validated. A variety of new results for vibration problems of the FG doubly-curved shells and panels with different elastic restraints, geometric and material parameters are presented. The effects of the elastic restraint parameters, power-law exponent, circumference angle and power-law distributions on the free vibration characteristic of the panels are also presented, which can be served as benchmark data in the research and the actual production process.

1. Introduction

The functionally gradient moderately thick doubly-curved shells and panels have obtained the widespread application such as architectural structures, hydraulic structures, containers, airplane, missiles, ships and instruments due to their excellent performance of the smooth and continuous mechanical behavior, light weight and form efficiency. Thus, it's of great significance to have a good understanding of the vibration behaviors of functionally gradient moderately thick doubly-curved shells and panels, which can provide some useful results for the designers and engineers to avoid the unpleasant and inefficient work, and the structurally damaging resonant in the design process.

So far, a lot of research work has been done for the vibration analysis of functionally gradient doubly-curved shells and panels in the literature. Aköz and Özütok [1] researched the free vibration of parabolic and circular cylindrical shells with

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classical boundary conditions with the mixed finite element method. Xie et al. [2] performed free vibration of spherical and parabolic shells of revolution with arbitrary boundary conditions by the Haar Wavelet Discretization (HWD) method. Kang and Leissa [3] solved the free vibration frequencies and mode shapes of solid paraboloids and complete paraboloidal shells of revolution with variable wall thicknesses and classical boundary conditions using a new three-dimensional (3-D) method. Khatib and Buchanan [4] presented free vibration analysis for determining the free vibration of a paraboloidal shell of revolution including shear deformation and rotary inertia with classical boundary conditions including free, fixed and simply supported adopting the finite element method. Lee [5] applied the pseudospectral method to study the axisymmetric and asymmetric free vibration analysis of spherical caps with classical boundary conditions. Pradyumna and Bandyopadhyay [6] studied the free vibration of functionally graded curved panels with classical boundary conditions using the finite element formulation based on a higher-order shear deformation theory. Tornabene and his team [7–14] extended the generalized differential quadrature (GDQ) method for the free vibration analysis of functionally graded doubly-curved panels and shells of revolution with classical boundary conditions. Besides, other related research results with the layered composite, isotropic and anisotropic parabolic and circular panels can be seen in Refs. [15–30]. From the above literature review, it's not hard to be found that the available results for functionally gradient doubly-curved panels and shells of revolution are scarce. In addition, the computing accuracy and the application scope of boundary conditions are deeply affected by the reasonable shell theory and solution approach in view of the specific problem. The thin classical shell theory is only fit for the thin shells since it doesn't consider the effects of the shear and normal deformation along the thickness direction. So if the accurate solution for the thin shell's higher modes or the accurate solution for the moderately shell with relatively thickness is required, the moderately thick shell theory which contains the first-order shear deformation theory and higher-order shear deformation theory can be applied for it eliminates the deficiency of the thin classical shell theory. Although the two-dimensional shell theory can be used to obtain accurate enough results for the thin and moderately thick plate, it may be unsuitable when the thickness of the shell is beyond a critical value. Thus, to improve the defect of the thin classical shell theory and moderately thick shell theory, the thick shell theory is developed and it gets rid of the hypotheses and can be used to acquire the results with arbitrary precision. A lot of methods for the vibration analysis of circular and parabolic panels and shells of revolution have been reported in the above literature, such as the generalized differential quadrature method, Haar Wavelet Discretization (HWD) method, finite element method, pseudospectral method and so on. However, two main weaknesses of these methods exist: Firstly, the scope of the boundary conditions are almost limited to the classical boundary conditions like free, clamped, simply supported and their combination; secondly, these methods are hardly applied to the FG circular and parabolic panels and shells of revolution except Tornabene using the GDQ to do it. However, boundary conditions which the doubly-curved panels and shells of revolution encounter are not always classical and various unknown elastic boundary restrains exist in the practical application. Therefore, it is necessary and significant to propose a unified method which can efficiently solve the vibration problems for FG doubly-curved panels and shells of revolution subject to arbitrary classical and elastic boundary conditions and their combinations.

The improved Fourier series developed in 2000 by Li [31–33] is adopted to conduct the free vibration analysis of beams subjected to general boundary conditions. Then this method was fast accepted and generalized to various structures (i.e., beams, plates, shells and coupled structures) by the Du [34–37], Jin [38–66], Wang [67–72]. As a powerful method, the superiority of this approach has been adequately validated by comparing with other methods. The detailed theoretical analyses and mathematical principle can been seen in Refs. [31–33]. In this paper, a unified vibration analysis of the four-parameter functionally graded moderately thick doubly-curved shells and panels of revolution with general boundary conditions is presented based on the improved Fourier series solutions. The formulation of the theoretical model is based on the first-order shear deformation shell theory. The functionally graded panel structures consist of ceramic and metal in this paper. According to the general four-parameter power-law distribution, we assume that they all vary continuously in the thickness direction and study six types of power-law distributions for the ceramic volume fraction. The admissible function of the FG panels and shells of revolution is obtained by the improved Fourier series which is composed of a standard cosine Fourier series and several auxiliary functions whose introduction can remove any potential discontinuities of the original displacement and its derivatives at the boundaries. The solution is obtained by using the variational operation in terms of the unknown the expanded coefficients. The numerical results show the present solutions can enable more rapid convergence, higher reliability and accuracy. A variety of new vibration results for the FG doubly-curved shells and panels with elastic restrains as well as different geometric and material parameters are presented and the effects of the elastic restraint parameters, power-law exponent, circumference angle and power-law distributions on the free vibration characteristic of the panels are also presented, which can be served as benchmark data for the designers and engineers to avoid the structurally damaging resonant.

2. Theoretical formulations

2.1. Description of the model

As depicted in Fig. 1(a), an FG doubly-curved shell is the basic configuration of the considered problem. An orthogonal curvilinear coordinate system (φ, θ, z) is fixed in the reference surface of the shell which is taken to be at its middle surface. The displacements of the shell in the meridional φ, circumferential θ and radial z directions are denoted by u, v and w, respectively. As depicted in Fig. 1(a), the meridional angle φ is formed by the external normal $\mathbf{n}$ to the reference surface