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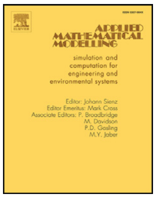

Three dimensional wave propagation in time-varying materials: A mathematical model based on the weak solutions of continuity in the moving property interface

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ABSTRACT

In this work, the elastic wave propagation through the moving property interface (MPI) in time-varying materials has been investigated. First, a general description of the MPI in time-varying materials has been given. Then, the elastic wave propagation through MPI has been systematically explored. In general, the wave propagation behaviors through MPI can be divided into two types depending on moving velocity of MPI and incident angle. For the first type, it has similar wave propagation behavior of that in static property interface, that is the reflection and refraction waves can be determined by the displacement and stress continuity in MPI. While, for the second type, it may have two transmitted waves, or only one transmitted wave or one reflected wave. Therefore, the transmitted waves or reflected wave cannot be solved by the commonly used continuous conditions, i.e. the displacement and stress continuities in MPI, due to the over constrained or under constrained problem. In order to deal with the wave propagation through MPI, a novel mathematical model is proposed based on the weak form of continuity in MPI. The results have shown that the wave propagation coefficients depend on not only the wave impedance radio and incident angle, but also the moving velocity of MPI. Besides, the transmitted and reflected wave frequencies and wavelengths are also significantly influenced by the moving velocity of MPI. When MPI catches up or exceeds the reflected or transmitted waves, the wavelength of the reflected or transmitted waves degenerates to zero, so that a shock wave is found in MPI and the displacements at the two sides of MPI are discontinuous. The wave propagation behaviors across MPI are further simulated via COMSOL Multiphysics software which shows very good agreement with the theoretical predictions. Moreover, the energy balance of SH waves in MPI is discussed. This work indicates that the novel mathematical model proposed in this paper is applicable to high dimensional wave phenomenon with mov-

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ing boundaries, and is capable for studying complex wave equations, especially for wave equations with moving boundaries.

1. Introduction

Wave propagations are common phenomena in a lot of fields, such as mathematics, physics, and mechanics, although their wave equations may be different depending on the controlling law of the wave propagation, constitutive relation of the materials, boundary conditions and so on. Besides, due to the very complex form of wave equation, it is also very hard to derive a simple and general conclusion to describe the wave propagation [1–5]. So, in this work we focus on studying the wave propagation in time-varying materials described by linear elasticity to obtain some general hints for wave propagation in such materials. Indeed, in recent years the wave propagation in time-varying materials has already been studied by Lurie [6] and Shui et al. [7,8]. In their works, the wave equations are described by linear elasticity with time-varying coefficients, like density or elastic modulus. Some novel wave propagation phenomena have been demonstrated in their works. However, three dimensional (3-D) wave propagation in time-varying materials has not been studied in previous literatures.

Wave propagation in time-varying materials has profound physical background. With the growing demand of intelligent materials or structures, it is of the primary importance to explore the effect of time-varying properties on the wave propagation behaviors, which is helpful to develop the wave propagation theory in time-varying materials and promote the applications of time-varying materials in acoustic and optical cloak. Here, the time-varying materials mentioned in this work are specifically referred to the materials that their properties can exhibit rapid varying with time. For the intelligent or active materials, their properties are usually the functions of external fields [9], such as temperature, electric field, magnetic field, light field, etc. So for such type of materials, the time-varying properties can be achieved by applying the external time-varying fields. For example, the magneto rheological fluids or magnetostrictive materials have time-varying damping when the time-varying magnetic fields are applied [10,11]. In addition, the controllable composite structures can also exhibit time-varying properties. For example, a phononic crystal with locally controlled connectivity has been proposed [12], which shows a few amount of adaptive material has remarkable effects on the phonon properties due to the switching of system periodicity. The structural composite with time-varying moment of inertia has also been proposed [8]. Anyhow, the time-varying properties can be achieved through precisely designing the active components of materials responding to external fields.

Because of the excellent performances caused by time-varying properties, intelligent materials have gained a lot of new applications [13–15]. Meanwhile, some novel structural composites have also been proposed based on the concept of time-varying properties [6,8,16–19]. Indeed, these studies have stimulated a new research field in elastodynamics for time-varying materials. Generally, two types of method (i.e. frequency domain analysis method and wave propagation method) are widely used to study the elastodynamics of traditional materials. However, for the time-varying materials, wave frequency isn't invariance during propagation which makes the frequency domain analysis method inapplicable. Thus, in this work the wave propagation method is used to study the wave propagation behaviors in time-varying materials. Indeed, the elastic wave properties in time-varying materials have been studied by a series of work, including the linear elastic waves in transient materials [17,20,21] and in a special time-varying structure with periodic transient properties [22,23]. Elastic wave propagation in a special time-varying structure with one dimensional (1-D) moving properties interface (MPI) has been classified [6,7]. Besides, the subsonic reflection and transmission coefficients of 1-D elastic waves at the 1-D MPI have also been deduced [16]. However, the previous studies of the elastic wave propagation in time-varying materials with MPI mainly focused on the simple 1-D case. A few works aim at the elastic wave propagation in 3-D time-varying materials. Therefore, in this work we first propose a general description of the MPI in 3-D time-varying materials, and then study the linear elastic wave propagation behaviors in such materials.

The rest of the paper is organized as follows. In Section 2, a general description of the 3-D MPI is proposed and in Section 3, the 3-D propagation of plane waves is simplified to a 2-D situation by choosing proper coordinate system. The reflection and refraction law, wavelength and frequency of linear elastic waves at the two sides of MPI, are obtained herein. In Section 4, the SH wave is taken as an example to study the wave propagation characteristics across MPI. It is found for some cases the transmitted and reflected waves cannot be solved by the commonly used continuities in MPI. Therefore, a novel mathematical model by considering weak solutions of continuities in MPI is proposed, based on which the propagation laws are analytically obtained. Besides, numerical verifications are carried out via COMSOL Multiphysics software [24] which agrees well with the theoretical predictions. In Section 5, the mechanical energy balance of SH waves in MPI is discussed and an extended moving property domain model (MPD) is proposed. By introducing MPD and viscous damping, the flaw of the infinite energy of shock wave in MPI is avoided. Finally, closing remarks are given in Section 6.

2. Description of MPI in time-varying materials

As the properties varying is caused by the time-varying external fields, it seems that the properties of time-varying materials are just a function of time. However, as a number of factors cannot be ignored, such as the edge effect, device-