Applied Mathematical Modelling 48 (2017) 451–475

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Dynamic analysis of short and long journal bearings in laminar and turbulent regimes, application in critical shaft stiffness determination

Farzad Hemmati[a,*], Mohammad Miraskari[a], Mohamed S. Gadala[b]

[a] Mechanical Engineering Department, University of British Columbia, 2054-6250 Applied Science Lane, Vancouver, BC V6T 1Z4, Canada
[b] Mechanical Engineering Department, Abu Dhabi University, Abu Dhabi, United Arab Emirates

ARTICLE INFO

Article history:
Received 8 November 2016
Revised 24 March 2017
Accepted 5 April 2017
Available online 13 April 2017

Keywords:
Linear and nonlinear stability analysis
Hopf bifurcation theory
Short and long journal bearings
Constantinescu's and Ng–Pan–Elrod turbulent models
Critical shaft stiffness

ABSTRACT

Linear and non-linear stability of a flexible rotor-bearing system supported on short and long journal bearings is studied for both laminar and turbulent operating conditions. The turbulent pressure distribution and forces are calculated analytically from the modified Reynolds equation based on two turbulent models; Constantinescu's and Ng–Pan–Elrod. Hopf bifurcation theory was utilized to estimate the local stability of periodic solutions near bifurcating operating points. The shaft stiffness was found to play an important role in bifurcating regions on the stable boundaries. It was found that for shafts supported on short journal bearings with shaft stiffness above a critical value, the dangerous subcritical region can be eliminated from a range of operating conditions with high static load. The results presented have been verified by published results in the open literature.

1. Introduction

The demand for high pressure injection of natural gas in underground has led to the design of multistage compressors. In 1970s many of such compressors were suffering from violent sub-synchronous whirl [1], a form of self-excited instability. In turbo-machinery, instabilities are characterized by whirling of the rotor bearing systems at frequencies other than the rotating frequency of the shaft. While large amplitudes of sub-synchronous vibration do not occur frequently, they can appear at certain operating conditions and can lead to high amplitude and destructive vibrations. If the operating speed of rotors exceeds a threshold speed known as "threshold speed of instability", the rotor-bearing system becomes unstable in oil whirl/whip which is characterized by sub-synchronous whirling [2]. Linearized stiffness and damping coefficients (dynamic coefficients) are used as basis for stability analysis of the rotor bearing systems. In the linearized analysis, bearing dynamic coefficients are evaluated at the equilibrium position of the journal.

While the load–displacement curve of a journal bearing is evidently nonlinear, the bearing behavior and stability can be characterized by means of linear dynamic coefficients if certain conditions are met. It is known that the "local" stability of a non-linear system and its linearized counterpart are essentially the same even though their stability type could be different. By local, one means that the current operating condition is close enough to an operating equilibrium point, a condition that is also vital for validity of utilizing linearized bearing coefficients to represent the journal force in rotor-bearing analysis. Choy et al. [3] calculated nonlinear bearing stiffness coefficients and showed that for displacements sufficiently far away from the equilibrium position, oil film forces will exhibit nonlinearities. Andres and Santiago [4] experimentally determined

---

[*] Corresponding author.
E-mail addresses: farhemmati@gmail.com (F. Hemmati), m.miraskari@alumni.ubc.ca (M. Miraskari), gadala@mech.ubc.ca (M.S. Gadala).

http://dx.doi.org/10.1016/j.apm.2017.04.013

Nomenclature

L	bearing length
M	reduced mass of the rotor
C	radial clearance (m)
R	journal radius (m)
ε	eccentricity ratio
O	center of bearing chamber
O_j	center of the journal inside the chamber
g	gravitational constant (m/s^2)
ω	shaft rotating speed (rad/s)
$\bar{\omega}$	dimensionless shaft rotating speed ($\omega\sqrt{M/K_s}$)
$\bar{\omega}_s$	critical speed of the shaft
ω_w	whirl rotating speed (rad/s)
k_θ	circumferential turbulent coefficient
k_z	longitudinal turbulent coefficient
ρ	fluid film density (kg/m^3)
W	load per bearing ($\frac{Mg}{2}$)
$\bar{\Gamma}$	stability parameter ($\frac{C}{W}M\omega_s^2$)
$\bar{X}_i$	dimensionless coordinate in horizontal and vertical directions ($\frac{X_1}{C}$)
$\bar{K}_{ij}$	Cartesian-coordinate dimensionless stiffness coefficients ($\pi(C/R)^3/\mu\omega L$) K_{ij}
S_z	non-dimensional shaft stiffness coefficient, ($\frac{C}{W}$)K_s
$\bar{F}_{r,t}$	non-dimensional forces, $F_{r,t}(\frac{\pi C^2}{\mu\omega LR^3})$
F_R	radial fluid force (N)
F_T	tangential fluid force (N)
F_x	radial fluid force in x direction (N)
F_y	radial fluid force in y direction (N)
F_x^t	total force on bearing in x direction (N)
F_y^t	total force on bearing in y direction (N)
S	Sommerfeld number ($\frac{\mu\omega LR^3}{\pi WC^2}$)
K_s	shaft stiffness
Ω	whirl frequency ratio
$\overline{Re}$	mean Reynolds number ($\frac{\rho R\omega C}{\mu}$)
Re^*	local Reynolds number ($\frac{\rho R\omega h}{\mu}$)
μ	fluid film (lubricant) viscosity (Pa s)
$\bar{k}_{ij}$	polar dimensionless stiffness coefficients ($\pi(C/R)^3/\mu\omega L$) k_{ij}
$\bar{c}_{ij}$	polar dimensionless damping coefficients ($\pi(C/R)^3/\mu L$) c_{ij}
θ	circumferential coordinate
φ	attitude angle
h	oil film thickness, $C(1+\varepsilon\cos(\theta))$
P	oil film pressure (Pa)
$\bar{P}$	dimensionless oil film pressure ($(\frac{C}{D})^2\frac{8\pi}{\mu\omega}$)
a_i, $\hat{a}_i$	turbulent constants
$\bar{C}_{ij}$	Cartesian-coordinate dimensionless damping coefficients ($\pi(C/R)^3/\mu L$) C_{ij}

the dynamic coefficients under high dynamic loads with large orbital motion of up to 50% of the bearing clearance. Their results are in good agreement with analytical linearized coefficients. Meruane and Pascual [5] estimated the linear and nonlinear bearing coefficients under large orbital motion and even during oil whirl. They showed that the linearized analytical coefficients agree reasonably with linear coefficients estimated numerically considering a nonlinear model and under large orbital motion. Small variation between the linear and nonlinear model were reported provided that the operating speed is kept below the instability threshold speed. Muzakkir et al. [6] concluded, based on experimental results, that high viscosity lubricants for heavily loaded slow-speed journal bearings, in laminar regime flow, would improve bearing stability; however, no stability boundary region were provided. Lahmar et al. and Singh et al. [7,8] provided pressure distribution and bearing coefficients for thrust and compliant journal bearings under laminar flow assumption and no discussion were provided at high velocities were fluid film behaves turbulent.