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A modified numerical manifold method for simulation of finite deformation problem

Wei Wei [a], Qinghui Jiang [a,b,*]

[a] School of Civil Engineering, Wuhan University, Wuhan 430072, PR China
[b] School of Civil Engineering and Architecture, Nanchang University, Nanchang 330031, PR China

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ABSTRACT

With many people contributing to its modifications and advancements, the numerical manifold method (NMM) is now recognized as an efficient tool to solve the continuum–discontinuum coupling problem in geotechnical engineering. However, false solutions have been found when modeling finite deformation problems using the original NMM. Based on the finite deformation theory, a modified version of NMM is derived from the weak form of conservation of momentum and the corresponding traction boundary condition. By taking the dual cover system as the displacement approximation, the governing equations of the modified NMM are formulated. A comparison of the governing equations of the original NMM and modified NMM illustrates the reason that the original NMM is not suitable for simulation of finite deformation problems. Three numerical examples are investigated to verify the capability of proposed method to predict static and dynamic finite deformation response. Numerical results show that the modified NMM eliminates the errors caused by large rotation and large strain, and obtains a good agreement with analytical solutions and the finite element method.

1. Introduction

The numerical manifold method (NMM), which was initially proposed by Shi [1,2], is a combination of the finite element method (FEM) and discontinuous deformation analysis (DDA) [3]. It provides a unified way to solve continuous and discontinuous problems in geotechnical engineering. Over the past few decades, numerous modifications and enhancements [4] has been done to improve this method.

Owing to its advantage in solving continuous and discontinuous deformations, most NMM researchers concentrated on crack propagation problems. Tsay et al. [5] firstly employed NMM to investigate crack growth path using local remeshing technique. Chiou et al. [6] applied the NMM to predict mixed mode crack growth combined with the virtual crack extension model. Li et al. [7] developed the meshless manifold method to solve two-dimension crack problems. Terada et al. [8] extend the finite cover method (FCM), an alias of NMM, to analyze the progressive failure processes using the cohesive fracture zone model in heterogeneous solids and structures. Kurumatani and Terada [9] extended the FCM to simulate the crack growth of quasi-brittle heterogeneous solids by introducing 'multi-cover-layer modeling'. Ma et al. [10] adopted NMM to model the complex crack problems such as multiple branched and intersecting cracks. Zheng and Xu [11] discussed the rank deficiency, the integrals with singularity of 1/r, kinked cracks and the mesh independency

* Corresponding author at: School of Civil Engineering, Wuhan University, Wuhan 430072, PR China.
E-mail address: jqh1972@whu.edu.cn (Q. Jiang).

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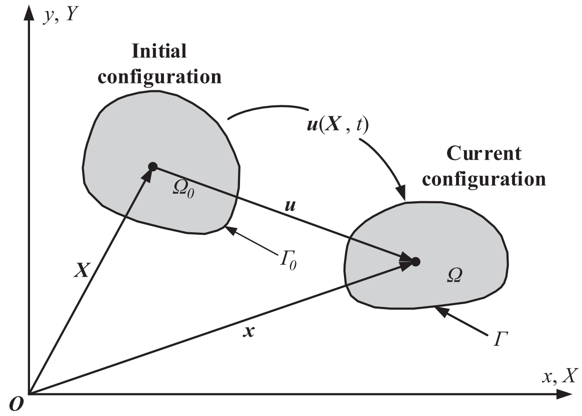

Fig. 1. Configurations of body.

of NMM in the simulation of crack growth using NMM. Zheng and Liu [12] reduced the problem of growth of multiple cracks to a nonlinear complementarity problem (NCP) and solved it by the projection-contraction algorithm PCA

Using the fracturing algorithm based on the Mohr–Coulomb criterion with a tensile cutoff, Ning et al. [13], An et al. [14] and Wong [15] made preliminary studies to model progressive failure in rock slopes and analyzed the stability of slopes. By introducing a rock bolt element in the NMM, Wu and Wong [16] investigated the rock block falling case in reinforced tunnels. Wei et al. [17] established a numerical model for modeling the bolts across the joint surfaces in the NMM. Zhang et al. [18] proposed a method combining NMM and graph theory to assess the slope stability and obtain the safety factor and the critical slip surface of slopes. By developing a sequential excavation algorithm, Tal et al. [19] applied the improved NMM to analyze the stability of tunnels in the Jingping hydropower project in Sichuan Province, China. Except for the applications to structures and solids, the NMM is also employed to conduct seepage analysis [20–22], the analysis of thin plate bending problems [23] and so on.

To improve the accuracy of solutions, high order approximations of field variants in the NMM are constructed [24,25], which might lead to the issue of linear dependency (LD). An et al. [26] dissected the origin of LD and proposed an approach to predict the rank deficiency of the global stiffness matrix. Ghasemzadeh et al. [27] developed dynamic high order NMM without linear dependence problems. Recently, Tian and Wu [28] proposed an effective high order scheme avoiding the linear dependency problem in the XFEM setting, which is also applicable to the NMM. In the case of improving contact algorithm, Yang and Zheng [29] proposed a direct approach to treatment of contact in Numerical Manifold Method.

So far the NMM has been successfully applied to many aspects. However, the simulation of finite deformation problem using NMM is seldom reported. Fan and Zheng [30] proposed a new S-R-D Based NMM to simulate small or large deformation together with impact/contact. In this present study, a modified version of NMM based on finite deformation theory is proposed. By considering the conservation of mass, the derivation of discrete equation of the modified NMM started from the weak form of conservation of momentum and the corresponding traction boundary condition. Then the two cover systems were introduced to construct the approximation of displacement field. To illustrate the reason that the original NMM for calculating the finite deformation problems is inaccurate, the comparison of governing equations of the original and modified NMM is demonstrated. Finally, three numerical examples are investigated to verify the accuracy and validity of proposed method.

2. Governing equations

As shown in Fig. 1, we consider a body which occupies a domain Ω_0 with a boundary Γ_0 in initial state at a time $t=0$. The domain Ω_0 is called the initial configuration and the position of a material point in this configuration is denoted by $\mathbf{X}$. when forces act on the body, it starts movement and deformation. The body occupied a domain with a boundary in a certain time t. Similarly, the domain Ω is called the current configuration and the position of a material point in this configuration is denote by $\mathbf{x}$. Therefore, the motion of the body is denoted by

$$\mathbf{x} = \mathbf{u}(\mathbf{X}, t) + \mathbf{X}, \tag{1}$$

where, $\mathbf{u}$ is the displacement of the material point $\mathbf{X}$.

The governing equations for the body is given as follows:

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} = \rho \ddot{\mathbf{u}} \tag{2}$$

$$\boldsymbol{\sigma}^{\nabla} = \mathbf{C}^{\nabla} : \mathbf{D} \tag{3}$$

$$\mathbf{D} = sym\left(\nabla \cdot \dot{\mathbf{u}}\right), \tag{4}$$