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An improved high precision measuring method for shaft bending deflection

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ABSTRACT

The circular-section columnar parts, such as shaft, rod and pipe etc., are widely used in all kinds of machines. The bending deformation of such parts will seriously affect the assembly precision and working performance of the machines. A high efficiency and precision measuring method for bending deflection is needed. With a classic contact-detection mechanism at low cost composed of a displacement sensor, a lever and a shaft supporting device, a high precision measuring algorithm is proposed and deduced after strict mathematical analysis in this paper, and which is a closed analytic algorithm and will not produce any principle error. The parameters sensitivity auditing is carried out and the result shows the measuring method has high adaptability. Furthermore, the corresponding experimental calibration method of the current measurement technology is provided, and an experimental validation on the bending deformation of a particular shaft is carried out. The result shows in depth the proposed method has high accuracy.

Notation

A, B	x and y coordinates of the rotating center
A_0, B_0	x and y coordinates of the geometric center
C, r, E	structural size of the lever mechanism
e, e_0	shaft section eccentricity
k_0	lever proportional coefficient
K_A, K_M	conversion coefficients for calibration
R	shaft section radius
z_c	actual measured value of the sensor
z_A	fluctuating amplitude of the sensor indicating value
z_M	sensor mean indicating value
z_A^e	fluctuating amplitude in experiment
z_M^e	mean value in experiment
z_A^L	theoretical fluctuating amplitude
z_M^L	theoretical mean value
ε	eccentricity deviation
ϑ	tilt angle between the lever and x axis

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ϑ_0 tilt angle for the equilibrium position
φ eccentric angle of shaft section

1. Introduction

As widely-used important components in machines, shaft, rod, pipe and other circular-section columnar parts have many functions and applications, such as transferring movement and power, supporting rotating parts, bearing loads, and so on. The bending deflection of such parts will severely affect the assembly precision and working performances of the machine in the way of centrifugal vibration, operational accuracy, even reliability and security [1]. In order to test, control or straighten the bending deformation of shaft parts [2] induced in the process of manufacturing or using, the bending deflection measuring is very important and necessary [3,4]. And in order to meet the requirements for automatic mass production, a high efficiency, high precision and automatic measuring method for bending deflection is needed.

Manual measuring bending deflection of a shaft with micrometer and V type block is clearly unable to meet requirement of modern high efficiency production. So Minoru et al. [5] has developed a transmission-type position sensor for the straightness measurement, but it could only been applied for a large structure. Rana et al. [6] has proposed a non-contact method for rod straightness measurement based on transmitting the laser light from a laser source and detecting on the other end with quadrant laser sensor, but the ranges of the length and the bent of the measured rod were limited. Feng et al. [7] has used a single-mode fiber-coupled laser module for straightness measurement to enlarge the measuring range. In fact, bending deflection of shaft, pipe and rod parts can be evaluated by the cross-section eccentricity. Schalk et al. [8] has presented a pipe eccentricity measurement system based on laser triangulation. However, the laser collimator technique has been difficult to guarantee the measurement precision because of the influence produced by laser beam drift, reflection of light and air turbulence so on; and its larger size and more expensive price have prevented it from being widely used as a testing system. Lu et al. [9] and Derganc et al. [10] have tried to use machine vision technique to measure bending deflection of rotating parts, but the measuring capability is determined by the development of image processing ability.

In present work, basing upon a classic measuring system at low cost composed of a displacement sensor, a lever and a shaft supporting device, a contact measurement technique for shaft bending deflection was made, which can avoid the concentric alignment error in some optical measurement techniques. And a high precision algorithm to obtain the deflection was presented, which was investigated by analyzing the accurate analytic relationship between the shaft cross-section eccentricity and the sensor indicating value. Sensitivity of the parameters in the algorithm was audited and a verification experiment was done on the effectiveness and high accuracy of the measuring method proposed.

2. Mechanism and model analysis

2.1. A contact-detection mechanism of bending deflection

A contact-detection mechanism of bending deflection is simplified as shown in Fig. 1. It is composed of a displacement sensor, a lever and a shaft supporting device. Since the bending deformation of the measured shaft can be described by the eccentricities at several cross-sections, this mechanism focuses on the measurement of section eccentricity. When the shaft rotates, the circumference of a measured shaft section will always contact with the measuring lever tightly. The eccentricity of the measured section can be transformed into the fluctuation of the indication value of the displacement sensor by the swing of the lever. If the mathematical relationship between the value of the sensor and the eccentricity is established, the eccentricity of the shaft section can be obtained by tracking the sensor value. Comparing with some no-contact methods using laser or machine vision technique, the current contact detection with the lever mechanism cannot been interfered by light and air and can better control the influence factors of measurement accuracy. Moreover, a small range displacement sensor can be selected because of the proportional reduction of the displacement signal, thereby the measuring mechanism cost is low.

As shown in Fig. 1, the rotating center of the lever mechanism is defined as the origin of Cartesian coordinates, recording as $O(0,0)$. And C, r, E are the basic structural dimensions of the lever mechanism. The radius of a measured cross-section can be noted as R. The rotating center of the tip can be noted as $O'(A,B)$, the geometric center of the section is noted as $O''(A_0,B_0)$. Obviously, $A_0 = A + e\cos(\varphi)$, $B_0 = B + e\sin(\varphi)$, where e and φ are the eccentricity and phase angle respectively. θ is the title angle between the lever and x axis, z is the sensor indicating value.

If the measured cross-section is not eccentric and the radius $R = B - E$, it is easy to deduce that the lever mechanism is always in horizontal position, i.e. θ is always equal to 0, and the indicating value z remains unchanged as the workpiece rotates, as shown in Fig. 1(a). If the measured section is eccentric, the indicating value z will fluctuate with the lever mechanism swinging during the workpiece rotating. But when the section radius $R = B - E$, the equilibrium position of the lever mechanism is in horizontal line; when $R \neq B\text{-}E$, the equilibrium position of the lever mechanism will deviate from horizontal line, as shown in Fig. 1(b) and (c).

According to the principle of plane geometry and trigonometric function, line L_0, L_1 and L_2 presented in the Fig. 1(b) can respectively be expressed as

$$L_0 : x = -r \tag{1}$$