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Modified differential transformation method for solving nonlinear dynamic problems

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ABSTRACT

A family of explicit higher order time integration algorithms is presented. The proposed techniques are based on the modified version of the differential transformation methods. First four members of this family are considered thoroughly. To improve the numerical properties of the DTM scheme, two parameters are introduced in the displacement and velocity extrapolations. In order to find the optimum values of the suggested parameters, many numerical attempts are made. The numerical effects of both modified and current DTM procedures are compared. To validate the performance of the new algorithms, comparison studies are accomplished with the well-known time integration methods in solving some linear and nonlinear dynamic problems.

1. Introduction

A major problem that structural engineers are faced with is solving the dynamic equation of motion in linear and nonlinear behaviors. One of the most efficient methods to solve this problem is the use of direct numerical integration algorithms. These techniques divide the time domain to finite and small time steps. Usually, some formulations are assumed to estimate the responses in each time step. To evaluate the accuracy of a time integration method, usually two quantities are determined, amplitude decay function (dissipation) and period elongation error (dispersion) [1]. The numerical errors due to dissipation and dispersion will result in a numerical damping of the structural response and shortening or elongating the natural period of vibration, respectively. The stability of an algorithm depends on the amount of error propagated from a time step to the next one, and if it grows after a while, the results will diverge from the exact solution.

Numerical schemes are classified into two major categories, implicit and explicit methods. Another minor one, called predictor-corrector, is existed as well. In an explicit algorithm unknown responses are independent of each other and could be estimated directly, while in an implicit procedure; the unknown responses of the system are dependent on each other and a system of equations must be solved in order to find them. Predicting the responses and improving their accuracies are the main duty of the third group of the time integration schemes. Each of these methods has its own advantages and disadvantages. The most important feature of the implicit techniques is their large stability domains. However, since the convergence of these algorithms usually requires an iteration procedure at each time step, they are often complicated and time consuming. So far, many efforts have been made to obtain less computationally expensive methods [2–5]. The well-known Newmark algorithm [6], HHT-α algorithm [7], WBZ-α scheme [8], generalized-α method [9], the family of IHOA methods [10], the algorithm presented by Rezaiee-Pajand and Sarafrazi in [11] and the family of MIHOA algorithms

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[12] are some examples of existing implicit methods. On the other side, there are explicit algorithms. The most important characteristic of these schemes is their independence of any matrix operation or iterative procedure at each time step. If the mass and damping matrices are diagonal, an explicit algorithm will be accomplished by vector operators [13,14]. So, compared to an implicit procedure, an explicit algorithm contains less computational effort. This advantage makes explicit schemes more efficient devices for problems with dominant high frequency modes [15]. The implementation of explicit procedures is also easier in parallel processing techniques and will lead to a reduction in calculation time. Despite the cost, the limited stability has created difficulties in applying the explicit techniques to structural dynamic problems. To overcome this drawback and as a simple solution, a small time step could be utilized during the analysis. Using explicit methods with a high order of accuracy can also be useful. There are different explicit methods available [16–18]. The generalized weighted residual approach [18], the *SSpj* method [19,20], the β_m algorithm [21] and the Hoff–Taylor scheme [22,23] are well known explicit procedures.

In many of the proposed algorithms, only the results of the previous step are used in the equations and data obtained from the previous steps will be ignored. In order to increase the efficiency of the numerical method, some researchers have used these data in the main equation. In 1996, Zhai proposed a simple explicit scheme that uses the accelerations of two previous time steps [24]. In the latter paper he also offered a predictor-corrector integration algorithm employing his proposed explicit method as a predictor and the Newmark implicit method as a corrector. According to Zhai, in spite of the existence of advanced techniques, the second order accurate Central Difference Method (CDM) is still the most popular explicit algorithm. He also noted that the reason for this popularity is the acceptable stability range of CDM, while other methods have limitations in their stability ranges. As a result, in the latter paper he tried to develop a method with a stability limit at least equal to the CDM one. In 2008 Rezaiee-Pajand and Alamatian offered a predictor-corrector algorithm using the acceleration of several time steps in the displacement and velocity formulations [25]. This technique has also been used by Keierleber et al. in an implicit algorithm [26]. In 2015, Rezaiee-Pajand and Hashemian optimized the weighted factors of Zhai formulations to achieve an unconditional explicit algorithm [27].

By adopting the idea of using data from the previous time steps and utilizing the concept of differential transform method a new family of the higher order explicit time integration procedures is presented in this paper. In 1986, Differential Transformation Method (DTM) that is based on the Taylor series was first proposed by Zhou. This investigator solved the linear and nonlinear initial value problems that appear in electrical circuits [28]. It is worth mentioning, DTM is a semi analytical-numerical approach that is useful for solving various differential equations. Utilizing this technique, high accurate or exact responses could be obtained in solving different types of the differential equations. With this method, the governing equation is reduced to a recursive equation that can easily be solved. Several researchers have benefited from DTM algorithm in solving linear and nonlinear equations, including equations of the beams, columns or vibration of plates. In 2011, Demirdag O. and Yesilce Y. studied the free vibration of a Timoshenko column using the differential transformation scheme [29]. Erturk et al. utilized the DTM approach to determine the solutions of nonlinear oscillators [30]. Liu Z. et al. took advantage of this method for free vibration analysis of uniform Euler-Bernoulli beam [31]. All obtained results in the recent years show that DTM is a reliable, fast converging and robust tool for solving many linear and nonlinear differential equations.

At first, the fundamental definitions and theorems of the DTM method are presented in this paper. Then, the idea for developing the DTM algorithm in solving the dynamic equation of motion is proposed. In fact, a procedure is offered to advance the DTM scheme, and a way of attaining the essential parameters for improving each member of the DTM family is described. Moreover, the numerical abilities of the suggested modified DTM procedures are compared with the previous versions. Finally, a few linear and nonlinear examples are solved to numerically evaluate the properties of the proposed formulations. Due to volume limitation of the article, only a few numerical experiences are presented here.

2. The differential transform method

2.1. Definitions

In this section, the basic definitions and theorems of the DTM will be stated.

Definition 1. If $f(x)$ be a given function of one variable defined at a point $x=x_0$, then the one-dimensional transformation of the k^{th} derivative of the (x), $F(k)$, is defined by

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)\bigg|_{x=x_0}. \tag{1}$$

Eq. (1) is called the transformed function of $f(x)$.

Definition 2. The differential inverse transform of $F(k)$ is defined as follows:

$$f(x) = \sum_{k=0}^{\infty} F(k)(x-x_0)^k. \tag{2}$$