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Novel reliability-based optimization method for thermal structure with hybrid random, interval and fuzzy parameters

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ABSTRACT

In this paper, novel reliability-based optimization model and method are proposed for thermal structure design with random, interval and fuzzy uncertainties in material properties, external loads and boundary conditions. Random variables are used to quantify the probabilistic uncertainty with sufficient sample data; whereas, interval variables and fuzzy variables are adopted to model the non-probabilistic uncertainty associated with objective limited information and subjective expert opinions, respectively. Using the interval ranking strategy, the level-cut limit state function is precisely quantified to represent the safety state. The eventual safety possibility is derived based on multiple integral, where the cut levels of different fuzzy variables are considered to be independent. Then a hybrid reliability-based optimization model is established with considerable computational cost caused by three-layer nested loop. To improve the computational efficiency, a subinterval vertex method is presented to replace the inner-loop and middle-loop. Comparing numerical results with traditional reliability model, a mono-objective example and a multi-objective example are provided to demonstrate the feasibility of proposed method for hybrid reliability analysis and optimization in practical engineering.

1. Introduction

Various uncertainties are unavoidable in practical engineering due to the aggressive environment factors, incomplete knowledge and inevitable measurement errors [1]. As a typical representative of uncertainty-based analysis and optimization, the system reliability has become a very important concept in both science and engineering [2]. The main techniques to quantify the uncertainties can be grouped into three categories: probabilistic method, interval analysis and fuzzy set [3]. In probabilistic framework, the uncertainties are modeled as random variables or stochastic processes by a great amount of sample statistical information, and the probabilistic reliability is considered as the most valuable issue in engineering [4,5]. Using parametric and non-parametric probabilistic models, Pellissetti et al. analyzed the reliability of a satellite structure with random material properties and external excitation [6]. Considering the huge computational cost caused by complex finite element model, a Kriging surrogate model was introduced as a substitute for structural probabilistic reliability analysis [7]. By shifting the boundaries of violated constraints to the feasible direction, Du and Chen proposed a new sequential optimization and reliability assessment method for structural probabilistic design [8]. Among the various

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numerical methodologies, Youn and Choi created a confident guideline in selecting the most desirable method for probabilistic reliability-based optimization design [9]. The application of probabilistic reliability requires sufficient information to construct the precise probability density functions of uncertain parameters, but the sample information is not always adequate in the early stage of numerical analysis and optimization design.

In order to overcome this shortcoming, the non-probabilistic convex set theory has been introduced to deal with the reliability problems, where the bounds of uncertain parameters are well-defined but sufficient information about probability distribution functions is missing [10–12]. Using the convex model, Kang and Luo presented a non-probabilistic reliability-based topology optimization method for the design of continuum structures undergoing large deformations [13]. Considering the correlation between different parameters, Jiang et al. constructed a multidimensional reliability model with various convex sets [14]. Under the interval model, Wang et al. presented a reliability optimization method for coupled structural-acoustic problem based on finite element computing results [15]. By means of first-order matrix perturbation theory, Chakraborty and Roy converted the interval reliability optimization problem into a deterministic problem [16]. In recent years, the fuzzy theory is also receiving widespread attention for reliability problems with subjective uncertainties [17–19]. Mon and Cheng addressed the fuzzy system reliability for components with different membership functions [20]. Cremona and Gao investigated the possibilistic safety index and defined it as the shortest distance from the coordinate origin to the failure surface in the infinity norm [21]. Guo et al. developed the fuzzy reliability model using interval theory, and made it more general to the various fuzzy systems with different types of membership functions [22]. By treating the cut levels of different fuzzy variables as independent random variables with uniform distributions, Li et al. proposed a novel fuzzy reliability model to measure the safety possibility from probability perspective [23].

For many practical engineering problems, different kinds of uncertain parameters may exist simultaneously [24–26]. Thus, to construct a hybrid framework which integrates the merits of different uncertain reliability models is desirable. Guo and Du proposed a sensitivity method by six indices for the reliability analysis with hybrid random and interval variables [27]. Based on performance measurement approach, Jiang et al. created two reliability analysis models for random uncertain structures with interval distribution parameters [28]. Under the hybrid probabilistic and interval model, Xia et al. proposed a reliability optimization method using probability theory, and provided a random moment method and an inverse mapping method to estimate the objective function and reliability index, respectively [29]. Qiu et al. presented several probabilistic interval reliability models for structural systems, which can efficiently predict the safety and failure intervals based on limited input information [30,31]. From the overall perspective, current research on hybrid reliability is mainly concentrated in the random interval problems, while the study on engineering systems with all three kinds of uncertainties is promising but mostly unexplored [32,33]. Besides, it should be pointed out that in traditional reliability analysis, only the ‘worst-case’ information where the limit state function is strictly greater than zero is considered. This operation makes the reliability assessment too conservative because of neglecting useful information in the transition state with extra safety possibility.

The purpose of this study is to develop novel model and method for hybrid reliability optimization design with random, interval and fuzzy parameters. This paper is structured as follows. The reliability problem with various uncertainties is firstly reviewed in Section 2. Using interval ranking strategy and integral calculation, a new reliability analysis method is proposed in Section 3. Subsequently, Section 4 creates a hybrid reliability-based optimization model, whose computational cost is considerable because of the nested-loop. In order to improve the computational efficiency, a subinterval vertex method with small computational cost is presented in Section 5, and converts the nested-loop problem into a single-loop one. In Section 6, two numerical examples are provided to verify the feasibility of proposed method, and the paper is concluded with a brief discussion at last.

2. Problem statement

In the practical engineering problems, due to the vaguely defined system characteristics, data imprecision, insufficient information and judgment subjectivity, various uncertainties in material properties, external loads and boundary conditions are unavoidable. In this study, the first three types of uncertainties are all considered. The uncertain parameters whose probability density functions can be defined based on sufficient sample data are modeled as l random variables

$$\boldsymbol{\alpha}^R = (\alpha_i^R)_l = (\alpha_1^R, \alpha_2^R, ..., \alpha_l^R). \tag{1}$$

The uncertain parameters whose lower and upper bounds can be determined by the limited information are quantified as m interval variables

$$\boldsymbol{\beta}^I = \left(\beta_i^I\right)_m = \left([\underline{\beta}_i, \bar{\beta}_i]\right)_m = \left(\beta_i^c + \Delta\beta_i^I\right)_m = \left(\beta_i^c + \Delta\beta_i\delta_i^I\right)_m = \boldsymbol{\beta}^{\mathbf{c}} + \Delta\boldsymbol{\beta}\delta^I, \tag{2}$$

where $\underline{\beta}_i$ and $\bar{\beta}_i$ are the lower and upper bounds of the interval variable β_i^I; $\beta_i^c = (\bar{\beta}_i + \underline{\beta}_i)/2$ and $\Delta\beta_i = (\bar{\beta}_i - \underline{\beta}_i)/2$ are called midpoint and radius, respectively; δ_i^I denotes the standard interval variable $\delta_i^I = [-1, 1]$.

The uncertain parameters whose membership functions can be provided by the expert opinions are described as n fuzzy variables

$$\boldsymbol{\gamma}^F = (\gamma_i^F)_n = (\gamma_1^F, \gamma_2^F, ..., \gamma_n^F). \tag{3}$$