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Non-periodic and chaotic response of three-multilobe air bearing system☆

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ABSTRACT

The dynamic response of a three-multilobe air bearing (TMAB) system is investigated for various values of the rotor mass and bearing number using a hybrid numerical scheme consisting of the differential transformation method (DTM) and the finite difference method (FDM). The validity of the numerical scheme is demonstrated by comparing the results obtained for the rotor center orbit under typical operating conditions with those obtained from the traditional FDM approach and a perturbation method, respectively. The dynamic behavior of the rotor center is then investigated for rotor mass values in the range of $1.0 \le m_r \le 16.0$ kg and bearing number values in the range of $1.0 \le \Lambda \le 5.0$. The phase trajectories, power spectra, bifurcation diagrams, Poincaré maps and maximum Lyapunov exponents show that the TMAB system exhibits a complex dynamic behavior consisting of periodic, quasi-periodic and chaotic motion at certain values of the rotor mass and bearing number. In general, the numerical results obtained in this study provide a useful insight into the dynamic response of TMAB systems. In particular, the results indicate the operating conditions which should be avoided in order to achieve a desirable periodic motion of the system.

1. Introduction

Three-multilobe air bearings (TMABs) consist of three pad sectors with fixed centers of curvature arranged such that they do not coincide with the bearing geometrical center. TMABs are economical to build and often offer a substantially better performance than full circular bearings. As a result, they are commonly used to support high-speed rotors in such applications as high performance NC and CNC machine tools and high speed turbopumps.

The static properties of elliptical and three-lobe bearings are relatively well known [1,2]. However, multilobe bearings exhibit self-excited whirl vibration under certain rotational speeds and loading conditions, and hence their dynamic response must be properly understood if the optimal bearing performance is to be achieved. Li et al. [3] established the linearized stability thresholds for four different multilobe journal bearing systems, namely elliptical, offset elliptical, three lobe and four lobe. For each bearing system, nonlinear transient analyses were performed at rotational speeds above and below the threshold speed, respectively, in order to compute the corresponding shaft orbits and bearing forces. In addition, a numerical fast Fourier transform approach was used to determine the frequency content of the nonlinear orbits. Davies [4] and Davies and Leonard [5] analyzed the static and dynamic performance of multi-recess journal bearing systems and applied

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a perturbation method to solve the unsteady recess flow continuity equation and obtain the dynamic recess pressures. The calculated pressure values were then used to obtain the dynamic load capacity of the bearings. Sawicki et al. [6] measured the dynamic force components of a hybrid bearing system experimentally and then confirmed the measurements theoretically by calculating the rotor dynamic coefficients. Ghosh and Satish [7] analyzed the dynamic recess pressures of multilobe bearings and obtained the loading capacity by solving the continuity equation under the assumption of an unsteady recess flow. The stiffness and damping coefficients of multi-lobe hybrid bearings were additionally investigated for various values of the offset factor, concentric pressure and eccentricity ratio. In general, the results showed that multilobe hybrid bearings exhibit a more steady rotational behavior than circular hybrid bearings.

Zhao et al. [8] investigated the complex dynamic behavior of a rigid rotor supported by an eccentric squeeze film. The results showed that sub-harmonic and quasi-periodic motions occurred at speeds of more than twice the system critical speed given larger values of the rotor unbalance and static misalignment, respectively. Adiletta et al. [9] examined the chaotic behavior of a rigid rotor supported by short bearings and found that the system performed sub-harmonic, quasi-periodic and chaotic motions at different values of the system parameters. Rashidi et al. [10] analyzed the effect of the system preload on the dynamic behavior of rigid rotors mounted in two, three and four-lobe noncircular gas bearing systems, respectively. In performing the analyses, the gas pressure distribution and equations of motion of the rotors were solved using the finite element method and the Runge–Kutta method, respectively. The results showed that the bearing systems exhibited a complex dynamic response consisting of periodic, sub-harmonic and quasi-periodic motions depending on the magnitude of the preload force and the values of the rotor mass and bearing number, respectively. Wang [11], Wang and Yau [12] and Wang et al. [13] using a hybrid numerical scheme comprising the differential transformation method (DTM) and the finite difference method (FDM) to analyze the bifurcation behavior and nonlinear dynamic motion of flexible rotors supported by a variety of air bearing systems, including relatively short spherical gas bearings, ultra-short gas bearings, and externally-pressurized double air film bearings. The results showed that the dynamic response of the bearing system (i.e., periodic, sub-harmonic, quasi-periodic and chaotic) was critically dependent on the values assigned to the rotor mass and bearing number.

In this study, the same hybrid method is employed to examine the nonlinear behavior of a rigid motor mounted in a three-multilobe (TMAB) system. The validity of the hybrid method for the considered TMAB system is demonstrated by comparing the results obtained for the rotor center orbit under typical operating conditions with those obtained from the traditional FDM approach and a perturbation method, respectively. The effects of the rotor mass ($1.0 \le m_r \le 16.0$ kg)and bearing number ($1.0 \le \Lambda \le 5.0$) on the dynamic response of the rotor orbit are then analyzed by reference to the corresponding dynamic orbits, phase trajectories, power spectra, bifurcation diagrams, Poincaré maps, and Lyapunov exponents, respectively.

2. Bearing system modeling

Circular bearing systems generally have a low load capacity under small eccentricity ratios. Moreover, they suffer severe instability if the system parameters (e.g., the rotor mass and rotational speed) are not properly assigned. Accordingly, non-circular bearings have attracted significant interest in recent years [7,10,14]. In general, the results have shown that the performance (e.g., load capacity and stiffness) of non-circular bearing systems can be tailored through an appropriate choice of the eccentricity ratio; with a higher ratio yielding an improved system stability and a decreased risk of non-linear motion.

2.1. Governing equations

Broadly speaking, non-circular bearing systems can be categorized as either offset, elliptic or multilobe. Of the three types of system, the offset-type has a high manufacturing cost and is thus seldom used in industry. The dynamic behavior of elliptic-type air bearing systems was examined by the present group in a previous study [14]. Thus, the current study focuses on the dynamic response of a multilobe air bearing system. Specifically, the study considers the TMAB system shown in Fig. 1, in which O_1, O_2 and O_3 are the arc centers of the three pad sectors, respectively, and O_J and O_B are the geometrical centers of the rotor and bearing. In addition, C_r is the bearing clearance and e_{JB} is the distance between the bearing and the rotor. Finally, e_1, e_2 and e_3 are the ellipticities of the three pads; defined as the distance between the respective arc center and the bearing center; and e_{J1}, e_{J2} and e_{J3} are the corresponding ellipticity ratios; defined as the ratio of the distance between the respective arc center and the rotor center. The ellipticity ratios for e_{JB} and e_{ri} ($i = 1, 2, 3$) are given respectively as:

$$e_{rJB} = \left(\frac{e_{JB}}{C_r}\right), \quad 0 \le e_{rJB} \le 1, \tag{1}$$

$$e_{ri} = \left(\frac{e_i}{C_r}\right), \quad 0 \le e_{ri} \le 1, \quad i = 1, 2, 3. \tag{2}$$

As shown in Fig. 1, the TMAB comprises three geometrical arcs with an angular amplitude of $\pi/3$, where the centers of these arcs form an iso-triangle around the bearing center.

For the first lobe area ($i = 1$):

$$e_{J1} = \left[{e_{rJB}}^2 + {e_{r1}}^2 - 2e_{rJB}e_{r1}\cos(\pi - \alpha_B)\right]^{1/2}, \tag{3}$$