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Mathematical modeling and simulation of the earth's magnetic field: A comparative study of the models on the spacecraft attitude control application

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ABSTRACT

In this paper, the Earth's magnetic field models which are widely used in spacecraft attitude control applications are modeled and extensively compared with a reference model. The reference model is obtained utilizing coefficients from the last generation of International Geomagnetic Reference Field (IGRF-12). The validity of this model is verified with the World Magnetic Model (WMM) in terms of intensity and direction of the field. The reference model is then used to evaluate lower-order and approximating models while the influence of effective parameters such as expansion order of modeling, orbit height, inclination, latitude and longitude on accuracy of modeling is investigated. The simulation results for several scenarios are presented and discussed. The linear and nonlinear transformations of the models from orbital frame to spacecraft body frame are compared for a wide range of attitude angles in order to investigate the sensibility and validity of linear transformation. Simulation of a spacecraft attitude control maneuver is performed to demonstrate the importance of the accuracy of the magnetic field model which is implemented in the attitude control system. The results indicated a meaningful increase in control effort when a simplified model was used. This research was aimed to investigate the borders of different geomagnetic field models and transformations for spacecraft attitude control applications. The presented results may lead to a proper choice of the Earth's magnetic field model based on the space mission requirements.

1. Introduction

In recent decades, there has been a growing interest in employing satellites in low Earth orbits (LEO). Not only a wide range of newly defined scientific objectives can be achieved by LEO satellites, but also significant mission cost reductions in such orbits have brought a great attention to them. This has led scientists to perform deeper investigations on all aspects of LEO missions and also to make revisions on topics that were quite known before.

An essential part of spacecraft for operation and achievement of mission goals is attitude control system (ACS). Extensive research has been done in the literature to improve the functionality of ACS in terms of accuracy and energy saving [1–8]. One of the conventional methods in spacecraft attitude control which is proved to be very effective and practical is the use of the Earth's magnetic field [4–11]. In this approach, a part or the whole required actuation is achieved by a set

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of electromagnetic coils producing the control torque in interaction with geomagnetic field [9–12]. Exploiting the Earth's magnetic field is especially applicable for LEO satellites since the field in such orbits has enough intensity to be used for attitude control. Furthermore, using magnetic coils is considerably less costly and less complex than other actuation methods used in ACS such as reaction wheels or thrusters [9,13]. In overall, it is evident that using geomagnetic field for satellite attitude control–alone or in cooperation with other methods– is widely considered for LEO missions. This is where the importance of accurately modeling the Earth's magnetic field becomes bold.

The Earth magnetic field can be analytically represented by a series of spherical harmonics including Gaussian coefficients and associated Legendre polynomials [14]. Hence, there are expansion models with different degrees which their accuracy depends on where they are truncated and how accurate the Gaussian coefficients of the series are measured. A first degree truncation constitutes a rather simple set of expressions named Dipole Model which models the Earth's magnetic field as a dipole. There are also models which make use of simplifying assumptions on dipole model to make them more convenient to use such as Centered Dipole Model and Simplified Dipole Model which will be further explained in Section 2. Second, third and fourth degree expansions are called Quadrupole, Octupole and Sedecimupole Models respectively. Expanding the harmonics to higher degrees increases the accuracy of modeling. However, the quality of Gaussian coefficients measurements confines us to the 13th degree according to the most recent generation of IGRF (IGRF-12) [15]. The 13th degree model which is valid till 2020 when the next version will be released is one of the most accurate and reliable models of the geomagnetic field currently available and is named Reference Model in this context.

The main purpose of this study is to give an assessment on various Earth's magnetic field models and reveal their limits and border of validity utilizing mathematical modeling and simulation techniques. To this end, a comprehensive evaluation of the Earth's magnetic field models is carried out considering the effects of important parameters such as expansion order of modeling and mission specifications such as orbital altitude and inclination. Next, the problem of transforming the geomagnetic field model from orbital frame to spacecraft body frame is considered. This transformation could be performed with either a linear method or a nonlinear one. The small angle assumption in spacecraft attitude control problem is shown to be a challenging topic in the literature [16]. There, we investigate how this assumption affects the results of geomagnetic filed model transformation.

In the first part of this paper, mathematics of geomagnetic field modeling required for generating the software codes used in simulations are described. Within this section, the selected models are introduced and transformations for describing the magnetic field in required coordinate systems are presented. Next, verification results for our reference model based on 12th version of IGRF are presented. The paper follows with the results of simulations and comparisons of the models for different scenarios. Effects of important factors such as orbital inclination, altitude and expansion order are depicted and discussed. Next, simulations are performed to evaluate the linear transformation of magnetic field from orbital frame to spacecraft body frame regarding various attitude angles. A simulation case study of an attitude control maneuver is considered for illustrating the importance of geomagnetic field model selection. Eventually, the conclusion of this paper is presented to briefly express the outcomes of this study.

2. Mathematical modeling of the earth's magnetic field

A rigid spacecraft can be described as a system of particles that their relative distances are fixed during the time. Attitude equation of such spacecraft is as follows [17].

$$\mathbf{T} = \dot{\mathbf{h}}_I = \dot{\mathbf{h}}_B + \omega \times \mathbf{h}_B, \tag{1}$$

where **T** denotes the external moments vector, **h** is the angular momentum vector, and $\boldsymbol{\omega}$ shows the angular velocity vector, the subscripts I and B refer to the inertial frame and body frame respectively. In derivation of the spacecraft attitude equations, external moments are shown as the sum of disturbance ($\mathbf{T_d}$) and control ($\mathbf{T_c}$) torques: $\mathbf{T} = \mathbf{T_c} + \mathbf{T_d}$ and the total angular momentum as the sum of rigid body ($\mathbf{h}$) and momentum exchange devices ($\mathbf{h}_w$) angular momentum: $\mathbf{h}_B = \mathbf{h} + \mathbf{h}_w$.

The magnetic control torque ($\mathbf{T}_c = \mathbf{T}_{mag}$) is produced by cross product of the magnetic dipole ($\mathbf{m}$) and the Earth's magnetic field vector ($\mathbf{B}$).

$$\mathbf{T}_c = \mathbf{T}_{mag} = \mathbf{m} \times \mathbf{B}. \tag{2}$$

The Earth's magnetic field, **B**, can be described as the negative gradient of a scalar potential function, V.

$$\mathbf{B} = -\nabla V. \tag{3}$$

The potential function, V, can be represented by a series of spherical harmonics [14]:

$$V(r, \phi, \theta) = R_e \sum_{n=1}^{N} \left(\frac{R_e}{r}\right)^{n+1} \sum_{m=0}^{n} [g_n^m \cos(m\phi) + h_n^m \sin(m\phi)] P_n^m(\theta), \tag{4}$$

where R_e is the equatorial radius of the Earth, g_n^m and h_n^m are Gaussian coefficients, r, φ and θ are the geocentric distance, co-elevation and East longitude from Greenwich respectively that can define any point in the space.

The set of Gaussian coefficients for use in the analytical models describing the Earth's magnetic field are called the International Geomagnetic Reference Field (IGRF). Every five years, a group from the International Association of Geomagnetism