Applied Mathematical Modelling 46 (2017) 465–475

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Dense underflow into a lake or reservoir—Sub-critical flow.

H.A. Omar, G.C. Hocking*

Mathematics and Statistics, Murdoch University, Perth, Western Australia, Australia

A R T I C L E I N F O

Article history:
Received 15 June 2016
Revised 23 December 2016
Accepted 17 January 2017
Available online 7 February 2017

Keywords:
Reservoir intrusion
Stratified fluid
Gravity current
Ideal fluids

A B S T R A C T

Steady two-dimensional flow of a dense stream down a slight embankment into a lake or a reservoir is considered. The inflowing water is separated from the ambient lake water by a density interface. This work follows on from earlier work in which the flows down a steep incline with a relatively high flow rate were considered. Here, the flow is slow and the entry angle is small, resulting in waves on the interface. The fluid is assumed to be of finite depth and the incoming channel makes an angle α to the horizontal. Limiting flows are found when the fluid separates at a stagnation point or alternatively when the waves reach maximum steepness. The regions in parameter space where such solutions are obtained are delineated for different flow conditions.

1. Introduction

Increased pressure on water resources in more arid regions of the world mean that it is more important than ever to understand the processes in storage reservoirs and lakes [1,2]. Better understanding leads to better management practices and hence more efficient utilization of vital potable water supplies. Cool water from mountain streams that enters reservoirs or lakes often has a lower density than the ambient lake water, leading to water flowing down the drowned river valley toward the dam wall.

Hebbert et al. [3] performed extensive field work and laboratory experiments to consider the dynamics of riverine inflow into a reservoir, and many of the resulting observations were incorporated into reservoir simulation models [2,4,5]. The path of water entering a stratified reservoir will depend on both the density of the inflow and the density of the ambient water. If the inflow is lighter, it will flow over the surface of the lake [6], while if it is heavier it will plunge to the bottom of the lake where it will flow along the bottom as a gravity current [7–9]. If it is at some intermediate density it will plunge downward until it reaches the level of neutral buoyancy at which point it will intrude horizontally [1,10].

Forbes et al. [10–12] examined stratified intrusions with constant density into three fluid layers where the top and bottom layers were assumed to be stationary and of finite depth, while the middle layer was in motion. Solutions were obtained with nonlinear, periodic waves on the intrusion layer and two branches of solution were found; at a higher speed both interfaces moved in phase, whereas they were out of phase at a lower speed. Solitary wave solutions were presented using both a full nonlinear method and KdV theory to produce weakly nonlinear solutions. The effect of stream bed slope, flow rate and density were considered in [12] and some interesting extreme solutions with looped waves were obtained. These extreme solutions are interesting but unlikely to be found in the field.

* Corresponding author.
E-mail addresses: hananomar42@yahoo.com (H.A. Omar), G.Hocking@murdoch.edu.au (G.C. Hocking).

http://dx.doi.org/10.1016/j.apm.2017.01.063

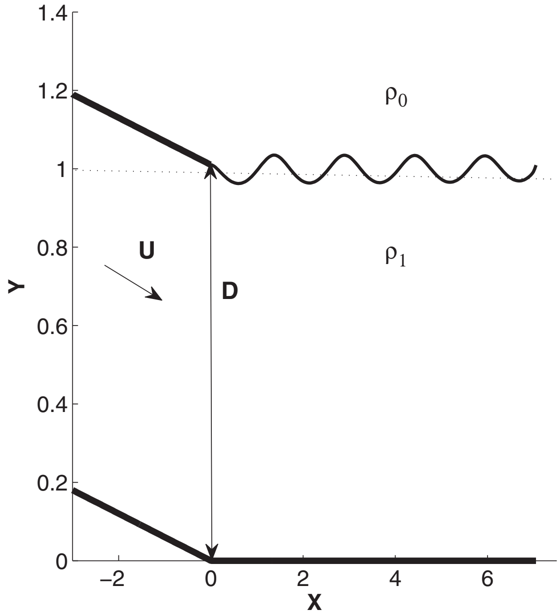

Fig. 1. Definition sketch of an intrusion flow from an angled structure with waves on the interface. The thicker sloping lines correspond to the solid inflow structure.

In a recent paper the authors, [13], used numerical and analytical techniques to examine the flow into a lake or reservoir when the flow rate is high, termed supercritical. Supercritical flow is defined in terms of the Froude number $F = \sqrt{U^2/(gH)}$ where U is the stream velocity, g is gravitational acceleration and H is the fluid depth. A Nekrasov type formulation was used in this paper [14], involving mapping of the unknown interface boundary to a known line in the complex plane. This formulation provided an exact solution for large Froude number ($F \to \infty$) and a very accurate, rapidly converging numerical scheme. In linear theory waves are not possible on a free surface or interface if the value of F is greater than unity ($F > 1$). They found that assuming irrotational flow of an incompressible fluid the high flows exist up to infinite flow rate and down to some limiting flow, with $F_{MIN} > 1$, in which the flow contains a stagnation point on the upper surface of the inflow structure above where it enters the lake. They also showed that in some cases there is a non-uniqueness in the solution domain, where two identical situations provide different solutions.

The related problem of flow past a semi-infinite flat plate has been studied by Vanden-Broeck [15–17], who found subcritical ($F < 1$) wave solutions on the interface downstream of a semi-infinite flat plate. McCue and Forbes [18] also considered such flows with a shear layer and found waveless solutions only when the height of the plate above the bottom is greater than the height of the shear flow. Maleewong and Grimshaw [16] considered the same problem with a plate depressed into the water using both nonlinear methods and KdV theory.

The present work is focused on intrusion flows along the bottom with a single interface when the flow rate is subcritical ($F < 1$). The interface between the layers of different density is assumed to be infinitesimally thin and as a first approximation viscosity and mixing at the interface are neglected. A different numerical formulation to the supercritical paper [13] was used in which the problem was solved completely in the physical variables. The reason for this was the direct nature of the method and the easier determination of the physical variables corresponding to a given flow.

The problems are also relevant to weir and waterfall flow where the interface is between air and water, in which case the model is more accurate. The assumption of the upstream flow being confined above by a rigid boundary is necessary to employ the techniques used in this work, but it is unlikely to alter the general behaviour of the flows.

Results are obtained at low flow rates across a range of parameter values to their limits. In some cases it is the elevation of the separation point that proves to be the limiting factor while in others it is the wave steepness. Out of interest we also consider the case of upward flow into the lake. These solutions, combined with those for downward flow, provide a complete picture of the solution domain under the assumptions considered here, and provide the foundation for a more detailed study involving the full flow equations.

2. Problem formulation

The steady, two-dimensional, irrotational flow in a layer of finite depth entering a lake as shown in Fig. 1 is considered. The layer has density ρ_1 and an interface that separates from an angled solid boundary as it enters the lake water. The background lake water has density ρ_0, where $\rho_0 < \rho_1$. The fluid is assumed to flow downstream in a homogeneous layer of