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A numerical study of rotation effect on the propagation of nonlinear internal solitary waves in the northern South China Sea

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ABSTRACT

In the paper, the rotation effect on the propagation of internal solitary waves (ISWs) in the northern South China Sea (SCS) is studied by applying non-dimensional analysis and a modified two-dimensional regularized long wave equation model. First, the contribution of each term in the model equation with rotation effect is estimated. It is shown that, the rotation effect is important once the scale of the propagating ISWs is larger than 100 km. Then, the model equation is differentiated and set up for the numerical simulation of ISWs in the northern SCS. Several numerical experiments are designed, and it is found that, the rotation effect mainly affects the symmetry of the ISW crestline, the amplitudes of the leading ISW and the north/south-ward extension crestline. With the rotation effect, the ISW crestline is not symmetrical, and the ISW amplitude decreases faster. For an incident depression internal soliton with an amplitude larger than 60 m, after the leading ISW propagates a distance of 330 km, the length of ISW crestline gets larger than 150 km, the contribution of rotation effect on the variation of leading ISW amplitude can reach more than 8%; the rotation effect and the bottom topographic effect on the variation of ISW amplitudes are opposite. The contribution of rotation effect on the variation of leading ISW amplitude decreases with an increase in amplitude of incident soliton, but it increases with the extension distance from the center of ISW crestline for the northern/southern crestlines of propagating ISWs.

1. Introduction

Internal solitary waves (ISWs) are ubiquitous in the world's oceans, they are very active in the northern South China Sea (SCS) which is located in the western edge of the Pacific. Since ISWs were found near the Dongsha Islands in the northern SCS by satellite images in 1970 s [1], many studies had shown that most of the observed ISWs there were in fact generated in the Luzon Strait which is several hundred kilometers away to the east [2–5].

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Many different dimensional numerical models were employed to study the propagation and evolution of ISWs. Among them, Korteweg-de Vries (KdV) equation and its modified equations models were commonly used to study the effects of nonlinearity, dissipation and shoaling topography, eddies etc., on the evolution of ISWs [6–8]. Rotation can also affect the propagation and evolution of ISWs, e.g, it is found that [9], due to the rotation effect, a pronounced westward bending of the northerly propagating ISW was visible in the satellite images of the Sulu Sea; [10] used a three-dimensional numerical model to investigate the reflection and diffraction of ISWs by an island, and showed that the rotation favored northward wave propagation in the region between the crossover point and the island, shifting the wave reconnection point behind the island northward; [11] used a fully-nonlinear, nonhydrostatic numerical model to investigate the propagation of ISWs in a rotating rectangular channel, and revealed that small-amplitude waves and sufficiently strong ISWs evolved differently under the action of rotation. In recent years there have been extensive studies of the effect of rotation on internal solitary waves within the scope of Ostrovsky equation (e.g, most notably in [12,13]). When the additional effect of rotation is included, the formation of solitary waves is inhibited [13], the trailing shelf formed behind the deforming ISW is considerably enhanced, which can lead to the formation of secondary wave packets resembling KdV-like undular bores [14]. In the northern SCS, the initial ISW can be long-lived so that the effects of the Earths background rotation and frictional effects need to be taken into account [12].

However, some questions remain unsolved, e.g., when does the rotation effect get important during the propagation of ISWs? Whether the rotation effects are different for different ISWs amplitudes or not? Moreover, most of the previous studies majored at the rotation effect on the variation of the ISWs along their propagation directions, whereas few are concerned at the rotation effect on the variation of the ISWs along their crestline extension directions (which are about perpendicular to their propagation directions). Without taking the rotation effect into account, [6] used a two-dimensional, regularized long-wave (RLW) equation model to study the propagation and evolution of ISWs in the northern SCS, including the wave-wave and wave-island interactions. There are lots of studies on soliton solution of RLW equation, e.g., we refer the interested reader to [15] for the meshless method using the Radial Basis Functions, and [16] for the Differential Quadrature and Globally Radial Basis Functions methods.

In this paper, on the basis of our previous work [6], first, we adopt the model equation by taking the rotation effect into account, and estimate the contribution of rotation effect by non-dimensional analysis; second, we differentiate the modified equation and develop a new numerical model, then apply it to the numerical study of rotation effect on the propagation and evolution of ISWs in the northern SCS.

The paper is organized as follows. In Section 2, the rotation effect on different scale ISWs is discussed by non-dimensional analysis; in Section 3, the setup of the model is presented; the numerical experimental results and discussion are described in Section 4, and Section 5 is conclusions.

2. Non-dimensional analysis

The two-dimensional regularized long wave equation model equation with the rotation effect is

$$\left[\eta_t + c\eta_x + \frac{1}{2}\alpha(\eta^2)_x + \frac{1}{3}\kappa(\eta^3)_x - \frac{\beta}{c}\eta_{xxt} + \frac{cQ_x}{2Q}\eta + \frac{C_D c\alpha}{3}\eta|\eta|\right]_x + \frac{c}{2}\eta_{yy} - \frac{f^2}{2c}\eta = 0. \quad (1)$$

Here, η is the displacement of interfacial pycnocline, c the first baroclinic modal phase speed, α the nonlinear parameter, κ the high-order nonlinear coefficient, β the dispersion parameter, C_D the drag coefficient of bottom friction, f the Coriolis parameter, x (eastward) and y (northward) are the coordinates in the two-dimensional Cartesian coordinate system, and t the time. Note that the rotation effect, the ninth term on the left side of Eq. (1), is now included in this model. This model is very closely related to the Ostrovsky equation [17] or the variable-coefficient extended Korteweg-de Vires equation [18,12]. It is regularized by replacing the η_{xxx}dispersion term with a term proportional to η_{xxt}, which could be referred to [19–21]. The model also accounts for a weak dependence on the second dimensional term proportional to η_{yy}, which is also well-known as the rotation-modified Kadomtsev–Petviashvili (KP) equation [12,17-21]. In fact, the same model as Eq. (1) (without the standard long wave regularisation) can be found in [22], which contains numerical studies devoted to internal waves without the effects of rotation and finiteness in the transverse direction in the South China Sea. The effect of slowly varying depth can be accounted for by including the weak additional term with Q in Eq. (1), and $Q = Q(x)$ is a known factor needed to ensure conservation of wave action flux $Q\eta^2$, which is described in a number of papers (e.g., see [12,14]). Here we set $Q = 2g(\rho_2 - \rho_1)c$as suggested by [14]. Thus, we have,

$$\left[\eta_t + c\eta_x + \frac{1}{2}\alpha(\eta^2)_x + \frac{1}{3}\kappa(\eta^3)_x - \frac{\beta}{c}\eta_{xxt} + \frac{c_x}{2}\eta + \frac{C_D c\alpha}{3}\eta|\eta|\right]_x + \frac{c}{2}\eta_{yy} - \frac{f^2}{2c}\eta = 0. \quad (2)$$

Although the characteristic wavelength of ISWs is often as long as several kilometers, ISWs can travel several hundred kilometers, and so can the extension distance of their crestlines [3,5]. In order to investigate the rotation effect on the propagation of ISWs, we apply the non-dimensional analysis to Eq. (2). The non-dimensional transformation of parameters is listed in Table 1, where the parameters with superscript '-' are non-dimensional.

Several non-dimensional parameters (Table 2) are introduced to represent the effects of different factors on the equation, e.g., ε_1, ε_2, ε_3, ε_4 and ε_5 represent the effects of low-order nonlinearity, high-order nonlinearity, dispersion, bottom friction and rotation, respectively.