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2D frames optimization. Criterion: maximum stability

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ABSTRACT

Structural analysis was one of the first disciplines to demand powerful computing tools. However, with current capacities of both calculus and manufacture and use of new materials, along with certain aesthetic conditions, it is possible to address problems such as the one presented in this article, whose aim is the optimal variation of any frame, so that few criteria are met, including stability. The problem is complex and must be solved numerically. This paper presents a formulation for solving the optimization problem, considering not only the buckling conditions but any other, such as allowable stress or limited displacement. The equilibrium of each beam in its deformed geometry is proposed under assumption of small displacements and deformations (Second Order Theory). The optimization problem is mathematically formulated to determine which values maximize the buckling load of the frame and numerically solved by sequential quadratic programming. Finally, for the optimal solution from the point of view of stability, the plastic collapse load is calculated. The plastic behavior is based on the bending moment and leads to sudden concentrated plastic hinges. Therefore, the structural stability is affected, which is checked during the loading process.

1. Introduction

Structural stability is an important area of great interest to engineering in which mechanical behavior of compression structural elements is studied. The first scientist that dealt with this problem was Galileo in 1638. In 1744, through his method of calculating variations, Euler obtained the differential equation of the elastic and the value of buckling load. In subsequent publications, he considered the case of variable section beams with axial load and other cases with axial load distributed along the length of the beam. In 1770, Lagrange [1] studied Euler's linearized equation and investigated the loads value that was higher than the former for Euler's buckling beam. In 1851, Clausen determined the optimal beam against buckling: the most stable column shape is one where the variation of the circular section along its length forms a curve similar to a cycloid [2–5].

Spillers and Levy [6] extended the buckling problem of a column to the optimal design of a plate bending against instabilities and, later on, to the study of the phenomenon of loss of stability in a symmetrical cylindrical shell along an axis [7]. However, a drawback to all these works is that their authors limited their optimal designs to only one restriction: a constant volume. However, in practice, the restrictions imposed by material resistance used or by displacement play an equally important role.

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Fu and Ren [8] resumed the above mentioned works, although they added the required stress restrictions, thus posing the problem of minimizing the volume of a column subject to a certain load by adjusting its geometric shape. The method used to solve this problem was the generalized reduced gradient, obtaining very favorable results.

There are also very recent references, employing the mathematical theory of functional analysis for obtaining the optimal shape against torsional buckling and/or bending from the conditions laid down by the principle of Pontriyagin [9]. As noted, the field of structural optimization has been attracting the attention of many researchers for some time. There are numerous recent publications [10–14] where various numerical techniques are presented in a multidisciplinary framework.

This work has focused on optimizing 2D frames, with an objective function posed in terms of stability; and as long as design restrictions or conditions are concerned, the most common ones may be considered: volume, stresses and displacements, but any restriction can be added.

On the other hand, the importance structural beam systems have in many engineering fields is well known, as well as their ability to withstand more load than that for which they were designed. This is partly due to the safety factors required by the regulations, as well as to their design being usually based on the elastic behavior rather than on the plastic regime. This would mean having a strength reserve that would achieve a more optimized structure design, meeting the real safety factor to certain overload, performing a vulnerability assessment of existing structures to certain ultimate limit states, or proposing efficient structure control systems. For this purpose, it is essential to have numerical models available that allow for adequate simulation of complex nonlinear phenomena occurring even in static regime, from which they can understand and quantify the limit behavior of beam structures. Limit state is defined as the situation where the structure or part of it ceases to behave appropriately under normal conditions (serviceability limit states) or collapses under severe accidental loads (ultimate limit state) [15]. This way, it would be possible to get to know or test the structure vulnerability and security against accidental actions, which is a topic of great interest in structural engineering.

2. Methodology

The effort in this work aims at developing a numerical method to optimally dimension beams in certain beam sections to form a 2D frame. To this end, among other possibilities, an objective function has been proposed in terms of the critical load factor, which is subject to restrictions on stress state, displacements, amount of material used, etc. This allows solving problems conditioned by instability phenomena, as well as others governed by the other bending behavior. Both the objective function, and the restrictions are generally not linear. This is why a method of Nonlinear Programming has been chosen as an optimization technique, namely Sequential Quadratic Programming algorithm (SQP, see Appendix) with the MATLAB® technical computing tool [16,17]. The programming code has been developed, so the choice of variables are simple and versatile, with the possibility of changing the objective function, and, finally, so it is easy to both eliminate and add new restrictions to the problem. In addition a global optimization algorithm has been used ensuring that the local maximum found is also global maximum (GlobalSearch Matlab's command is used for this purpose).

2.1. 2D beam model

The limitations imposed on the model are the following: application to planar structures with bisymmetric section beams, perfect plasticity (no hardening), achievement of the equivalent stresses by means of Von Mises' hypothesis and Navier–Bernoulli's beam model. Likewise, a proportional load state is adopted, in a way that all actions on the structure, except the weight and thermal load, increase in equal proportion with respect to their nominal values using the load factor (λ). Based on these assumptions the equilibrium equations, the compatibility and the behavior for the purpose of the beam element study are presented [11,13,18,19].

The 2D model used considers in their end sections (areas of beam bonding) semirigid nodes of longitudinal, transverse and rotational stiffness as given (see Fig. 1). This type of item has been chosen, instead of the classic rigid nodes, for two main reasons: first, because it allows the inclusion of any type of elastic hinge in a very simple manner. And second, because the variation in value of the rotational stiffness in a knot mesh during the loading process allows a simple way to introduce a plastic hinge.

2.2. Design parameters

With the intention of not complicating the method exposure and seeking an easy visualization of the results, it is suggested that the beam cross section depends on a single parameter h, which is the height of the double-T section. This parameter may vary over the spatial coordinate s of each beam $h(s)$. It is assumed that it has no loss of generality - a type of polynomial variation based on a finite number of parameters (a_i)-, while it is possible to suggest any other function:

$$h(s) = a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n. \tag{1}$$

These simplifications clearly illustrate the methodology. With this procedure the method is validated before extending its application to cases of more interest. The objective is to find the optimal height variation of the section at the ends of the columns and beams of frames, where, for numerical reasons, it is assumed that the law of variation is linear Fig. 2.