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Investigation on a rotating FGPM circular disk under a coupled hygrothermal field

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ABSTRACT

In this paper, the distributions of the temperature, moisture, displacement and stress of a functionally graded piezoelectric material (FGPM) circular disk rotating around its axis at a constant angular velocity under a coupled hygrothermal field are presented by a numerical method. The material properties of the FGPM circular disk are assumed to vary along the radial coordinate exponentially. First, the coupled hygrothermal field along the radius of a rotating circular disk is achieved by solving the coupled hygrothermal equations, and then the dynamic equilibrium is solved by utilizing the finite difference method. Finally, numerical results show the effects of functionally graded index, inner radius, angular speed and hygrothermal index on the hygrothermal behaviors of the FGPM circular disk. The results can be useful for the optimal design of rotating FGPM circular disks under a coupled hygrothermal field.

1. Introduction

Functionally graded piezoelectric material (FGPM) is a kind of nonhomogeneous composite material whose compositions and properties are designed varying continuously in space on macroscopic scale [1]. Due to the excellent performances in engineering applications, structural manufactures of FGPM have attracted wide and increasing attentions of both material and engineering researchers [2].

Investigations on various structural manufactures of FGPM are abundant in literature. Utilizing the finite element method, Ghorbanpour Arani et al. [3] carried out three-dimensional solutions for closed and open hollow spheres subjected to internal pressure and uniform temperature field. Dai et al. [4] presented an analytic solution to the axisymmetric problem of a long, radically polarized, FGPM hollow rotating cylinder by means of ordinary integration; and then using the infinitesimal theory of electromagnetothermoelasticity, Dai et al. [5] worked out dynamic electromagnetoelastic response of FGPM hollow sphere under coupled multi-fields. By applying finite difference method, Mao and Fu [6] carried out the researches of the nonlinear dynamic response and active vibration control of the FGPM plate. In the presence of supersonic aerodynamic loading, Rezaee and Jahangiri [7] studied the nonlinear, chaotic vibrations and stability of a simply supported functionally graded piezoelectric rectangular plate with bonded piezoelectric layer. Based on the three-dimensional theory of piezoelectricity, Wang et al. [8] investigated the axisymmetric bending of circular plates whose material properties varied along the thickness. Developing a direct displacement method, Li et al. [9] carried out a three-dimensional analysis of a

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Nomenclature

ω	rotating speed [r/min]
a,b	the inner and outer radii of the FGPM circular disk [m]
(r, θ, z)	the cylinder coordinate system
$c_{ij}(i, j = 1, 2)$	elastic coefficients [N/m^2]
$e_{1j}(j = 1, 2)$	piezoelectric coefficients [C/m^2]
ρ	mass density [kg/m^3]
g_{11}	dielectric constant [C^2/Nm^2]
$\alpha_i^T(i = 1, 2)$	thermal expansion coefficients [1/K]
$\alpha_i^C(i = 1, 2)$	moisture expansion coefficients [m^3/kg]
p_{11}	pyroelectric coefficient [C/m^{2}K]
s_{11}	hygroelectric coefficient [Cm/kg]
η	functionally graded index
r	radial coordinate [m]
L	equivalent diffusion coefficient of temperature [m^2/s]
ς	coupling coefficient due to moisture migration [K/$^\circ$M]
D	equivalent diffusion coefficient of moisture content [m^2/s]
λ	coupling coefficient due to heat conduction [$^\circ$M/K]
T	temperature distribution [K]
$B_{ij}(i, j = 1, 2)$	constants which relate to the thermal conductivity
$T_i(i = 1, 2)$	temperature on the inner and outer surfaces of the FGPM circular disk[K]
m	moisture potential [$^\circ$M]
$C_{ij}(i, j = 1, 2)$	constants which relate to the moisture diffusion
$m_i(i = 1, 2)$	moisture constants on the inner and outer surfaces of the FGPM circular disk
κ	thermal conduction coefficient [W/m K]
D_m	conduction coefficient of moisture content [kg/ms$^\circ$M]
c_p	heat capacity [J/kg K]
c_m	moisture capacity [J/kg$^\circ$M]
s	hygrothermal index
Λ	ratio of vapor diffusion coefficient to coefficient of total moisture diffusion
h_{LV}	heat of phase change [kJ/kg]
ξ	heat of absorption or desorption [kJ/kg]
u	radial displacement [m]
$\varepsilon_i(i = r, \theta)$	components of strain
$\sigma_i(i = r, \theta)$	components of stress [N/m^2]
D_r	radial electric displacement [C/m^2]
f_r	radial inertial force [N]
Φ	electric potential [W/A]

FGPM circular plate under tension and bending. Zhang and Zhong [10] presented a three dimensional exact solution for free vibration of FGPM circular plate.

In decades, multi-physical analyses of homogenous composites are easy to find in literature. Milazzo [11] presented a new one-dimensional model for the dynamic problem of magneto-electro-elastic generally laminated beams. Using a semi-analytical discrete-layer approach, Chen et al. [12] studied the free vibration of multilayered magneto-electro-elastic plates under combined clamped/free lateral boundary conditions. Developing an analytical method, Dai and Wang [13,14] obtained the transient response of magnetothermostress and perturbation of the magnetic field vector produced in an orthotropic laminated hollow cylinder subjected to thermal shock and a primarily uniform magnetic field. Recently, the multi-physical analysis of FGMs has attracted lots of interests of researchers. An analytical approach was employed by Li et al. [15] to solve the axisymmetric problem of FGM electroelastic cylinders with general variation of material properties using the Freedholm integral equation. Utilizing the Finite difference method and Newmark method, Dai and Rao [16] investigated dynamic thermoelastic behavior of a double-layered cylinder with a FGM layer under mechanical and thermal loads. Applying the power series method, Vel and Batra [17] presented an analytical solution for three-dimensional thermomechanical deformations of a simply supported functionally graded rectangular plate subjected to time-dependent thermal loads. Using a semi-analytical approach, Dai and Dai [18] achieved the displacement and stress fields in a FGM hollow circular disk, rotating with an angular acceleration under a changing temperature field.

Moreover, the multi-physical analysis of FGPM has attracted lots of researchers. Jamia et al. [19] investigated the problem of two collinear mixed-mode limited-permeable cracks embedded in an infinite medium made of a FGPM with crack surfaces subjected to electro-mechanical loadings. Employing a numerical Laplace inversion method, Akbarzadeh