Applied Mathematical Modelling 46 (2017) 299–311

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

On the flapping motion of a helicopter blade

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ARTICLE INFO

Article history:
Received 19 July 2016
Revised 24 November 2016
Accepted 17 January 2017
Available online 28 January 2017

Keywords:
Rotor blade design
Blade flapping motion
Periodic solutions
Helicopter vibration

ABSTRACT

It is well known that the flapping motion $\beta(t)$ of a helicopter blade can be modeled with an inhomogeneous linear differential equation with periodic coefficients. In this paper, we compare two methods that compute the Fourier coefficients of this motion. The first method provides an algorithm to numerically find the periodic solution $\beta(t)$. We provide a method to numerically compute the initial conditions of this periodic solution. Having this solution allows us easily compute as many Fourier coefficients as we want. The second method consists of either assuming that $\beta(t) = \beta_0 + \beta_{c1}\cos(\Omega t) + \beta_{s1}\sin(\Omega t)$ to find expressions for β_0, β_{c1}, β_{s1} or assuming that $\beta(t) = \beta_0 + \beta_{c1}\cos(\Omega t) + \beta_{s1}\sin(\Omega t) + \beta_{c2}\cos(2\Omega t) + \beta_{s2}\sin(2\Omega t)$ to find expressions for β_0, β_{c1}, β_{s1}, β_{c2}, β_{s2}. Or, in case more precision is needed, we assume $\beta(t) = \beta_0 + \beta_{c1}\cos(\Omega t) + \beta_{s1}\sin(\Omega t) + \cdots + \beta_{cm}\cos(m\Omega t) + \beta_{sm}\sin(m\Omega t)$ for some $m > 2$. It is interesting to point out that the expressions for β_0, β_{c1}, β_{s1} obtained by considering $\beta(t) = \beta_0 + \beta_{c1}\cos(\Omega t) + \beta_{s1}\sin(\Omega t)$ are different from the ones obtained when we consider $\beta(t) = \beta_0 + \beta_{c1}\cos(\Omega t) + \beta_{s1}\sin(\Omega t) + \beta_{c2}\cos(2\Omega t) + \beta_{s2}\sin(2\Omega t)$. The computation of blade flapping harmonics is extremely important for analyzing the helicopter harmonic content to gain a better understanding on how to achieve a better blade design. Blade design researchers who are unfamiliar with the first method will benefit immensely from using this method.

1. Introduction

Helicopter rotor blades generally operate at extremely high rotor rotational speeds required to generate the aerodynamic lift, thrust, and centrifugal forces to fly the helicopter. The blade flapping motion is obtained by balancing the blade's (i) aerodynamic thrust moment, (ii) gravitational force moment, (iii) centrifugal force moment, and (iv) inertia force moment with respect to its flapping hinge. Fig. 1.1 shows the respective forces associated with these moments. In hover, all blades are operating under the same aerodynamic flight conditions. The blade aerodynamic forces are symmetric among all blades. However, in forward flight, the situation is different. The helicopter blades encounter a higher rotor airspeed on the advancing side of the blades and a lower rotor airspeed on the blade's retreating side. An additional pilot input control is required to vary the blade's local angle of attack to redistribute the aerodynamic forces evenly on the rotor disc in flight. Therefore, the blades need to flap up and down in forward flight in order to balance the aerodynamic force distribution on the rotor disc in flight and to achieve a smooth ride.

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http://dx.doi.org/10.1016/j.apm.2017.01.055
0307-904X/

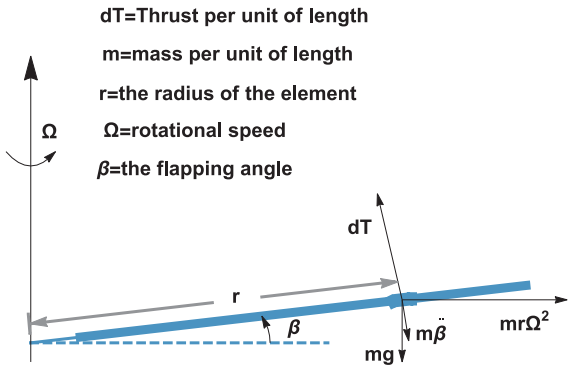

Fig. 1.1. Forces acting on a blade element.

Forward flight is a more difficult mathematical problem to solve than hovering, given the influence of the periodic coefficients on the flapping equations. It is impossible to find the exact solution for the nonlinear, time-varying differential equations. Peters and Hohenemser have used the Floquet Transition Matrix to find the largest eigenvalue of the linear flapping equations with periodically varying coefficients [1]. Hohenemser and Yin [2] and Biggers [3] have also studied the flapping rotors by using the method of multi-blade coordinates to transform the flapping equation from the rotating system into the fixed system. Meanwhile, Peters and Ormiston have investigated the flapping response of hingeless rotor using a generalized harmonic balance method [4].

Parkus has used a perturbation method based on the small parameter of advance ratio μ to investigate the flapping motion of a helicopter blade when disturbed by external gusts [5]. This may be the first time that a perturbation method has been used to handle the periodic coefficients in a rotor system. Johnson has investigated the stability of the flapping motion of a helicopter rotor blade in forward flight by using the perturbation method [6]. In order to obtain the analytical roots in forward flight, it is necessary to find a solution for time-varying equations that is valid over long time periods. Multiple time scales is the appropriate perturbation technique for such instances.

An improved method for solving the rotor blade flapping equation is described by Shutler and Jones that avoids the laborious computation required by an analytical solution [7]. The application of a second harmonic control on a helicopter rotor blade by Stewart causes a redistribution of the loading over the disc [8]. This can be used to increase the forward speed by delaying the stall on the retreating blade. The aerodynamic loading on the retreating and advancing sides could be reduced and compensated by increasing aerodynamic loading on the fore and aft sides. This will result in a more uniform distribution of incidence on the retreating side and postpone the stall limitation of the forward speed. Majhi and Ganguli have investigated helicopter blade flapping with and without small angle assumption in the presence of dynamic stall [9].

Because there are many dynamic components in a helicopter's rotor system design, vibrations are inherent during flight [10,11]. Helicopter vibration levels are significantly higher than those of the fixed wing aircraft. This vibration affects aircraft riding comfort and reduces component service life. Vibration also limits the helicopter's flight envelope during operation. Understanding how to reduce helicopter vibration is an important skill that an engineer needs in order to improve the quality of aircraft flight [12–16]. The main sources of helicopter vibration can be traced back to rotor blade mass unbalance, uneven blade airfoil contour that affects the blades' aerodynamics, and the excitation of blade running frequencies and their harmonics. Helicopter rotor blades produce vibratory aerodynamic forces and moments on the blades in forward flight. These aerodynamic forces emerge in the rotating system due to high rotational blade speeds, high induced flows, and asymmetric rotor air speeds during forward flight. Rotor blades are always in harmonic motion and higher harmonics are always important and difficult for engineers to predict. Higher harmonics also cause and amplify helicopter vibration [12,14–17].

During the full scaled SH-2F Composite Main Rotor Blade (CMRB) flight test, test engineers found that the newly developed blades introduced a significantly high 4/rev harmonic pilot seat area vibration in the fixed system [12,18,19]. This high vibration level was not acceptable for all pilots and engineers. Harmonic analysis of the flight test time history data indicated that the newly developed CMRB rotor blades produced very high 3/rev and 5/rev harmonics of out-of plane blade bending moments in the rotating system. These higher harmonics blade out-of-plane bending moments transmitted to the fixed system pilot seat area causing uncomfortable vibration levels. In order to reduce out-of-plane blade bending moment higher harmonic contents and to minimize pilot seat vibration, Wei and Jones decided to change the CMRB blade outboard tuning weight to increase the separation of blade 3/rev and 5/rev's natural frequencies and higher harmonics phase angles [12]. In addition, the servo-flap rotor blade inboard index angle was also changed to alter the blade's aerodynamic excitation. These modifications resulted in a significant change in both the blade 3/rev and 5/rev harmonic flatwise bending moment in the rotating system, which lowered the previously unacceptable vibration levels to acceptable levels for helicopter field services.