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Bayesian analysis of repairable systems with modulated power law process

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ABSTRACT

As a compromise between nonhomogeneous Poisson process and renewal process, the modulated power law process is more appropriate to model the failures of repairable systems. In this article, objective Bayesian methods are proposed to analyze the modulated power law process. Seven reference priors, one of which is also the Jeffreys prior, are derived. However, only four of them are taken into consideration because of their practicality. Propriety of the posterior densities considering the four reference priors is proved. Predictive distribution of the future failure time is obtained additionally. For the purpose of comparison, the simulation work and real data analysis are carried out based on both the objective Bayesian and maximum likelihood approaches, which show that the objective Bayesian estimation and prediction have much better statistical properties in a frequentist context, and outperforms the maximum likelihood method even with small or moderate sample sizes.

1. Introduction

In reliability theory, much attention has been paid to non-repairable system (unit, product) that can fail only once. The observation of a non-repairable system (unit, product) is known as the lifetime which may often be modeled by the exponential, Weibull or gamma distributions. However, most of the systems can be repaired, and the failure of a repairable system occurs repeatedly during a given period. The observations consist of a series of repair times or repair costs. Such an event is termed as a *recurrent event* in other fields like clinical study and software reliability. For more details, see Gail et al. [1], Dalal and McIntosh [2], and Therneau and Hamilton [3].

Stochastic point processes have been widely used to analyze lifetime data of repairable systems since they can characterize the failures by assuming they are distributed randomly in time. Nonhomogeneous Poisson process (NHPP) and renewal process (RP) are probably two of the most commonly used ones to describe the failure pattern. In some cases, the repairable system returns to the same state as before the failure after a repair. In other cases, the system becomes a new one after a repair. NHPP can depict the effectiveness of repairable system achieved by “same-as old” (minimal repair assumption) and RP can depict the effectiveness achieved by “same-as new” (perfect maintenance assumption). That is, both NHPP and RP perform well in the ideal world and are easy to handle. Nevertheless, in practice both “same-as old” and “same-as new” are two extreme failure pattern of repairable system, since repair can make a system perform better

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than it was before the failure occurrence but worse than a brand-new one. The NHPP seems pessimistic whereas RP is overly optimistic to fit the problem of repairable system. In public health, in addition, some diseases occur again and again after treatment and these two processes are obviously too simple to describe the machinery of life. Accordingly, models applied for analyzing recurrent event data must be capable of simultaneously delineating the relationship between current event and previous treatment and telling whether the individual experiences deterioration or improvement as it ages. These models must be inherently different from those for lifetime data. There are some scenarios that the NHPP model can be correctly used in reliability. For example, the system is complex and repair consists only in substituting the failed part with a new one which does little good or harm to anything else in the system. A complete discussion on repairable system and recurrent event data can be found in [4] and [5]. In this paper, we concentrate on discussing the recurrent event data from reliability, i.e. observations from repairable system, and assume that repair is provided as soon as the system fails.

To give a compromise between the nonhomogeneous Poisson process and renewal process, Lakey and Rigdon [6] introduced the modulated power law process (MPLP). It is an extension of NHPP with power or Weibull intensity which is often referred to as power law process (PLP). and also a special case of the inhomogeneous Gamma process defined by Berman [7]. For the modulated power law process, the failure probability at a given time t of the repairable system depends not only on the age of the system but also on the recent repair time. Thus, it can incorporate both renewal-type behavior and time trend. The MPLP is more suitable to model the repair or failure patterns in many practical settings in reliability. The MPLP can be used to analyze a wide range of point processes, such as the failure process of repairable systems, the duration dependence of business cycles [8], and stock market cycles [9]. Approaches used for exploring the MPLP so far are maximum likelihood method and subjective Bayesian method. Black and Rigdon [10] proposed an algorithm for deriving the maximum likelihood estimates for the model parameters. Asymptotic analysis served to give approximate confidence intervals and hypothesis tests for the parameters. Coverage probabilities of the asymptotic confidence intervals were strictly less than the nominal level in some settings. From the Bayesian perspective, Calabria and Pulcini [11] elicited non-informative and vague joint priors to proceed Bayesian analysis. In their paper, the non-informative prior was actually the independent Jeffreys prior which was obtained based on the independent assumption of parameters. Such assumption might be subjective and inappropriate, and make the "non-informative" prior informative. Moreover, propriety of the posteriors has not been proved in their article. Bayesian hypothesis tests for the parameters were further studied by Calabria and Pulcini [12]. Furthermore, Muralidharan [13] also presented various tests for the model parameters in the presence of nuisance parameters. Reliability of the ith process of MPLP was studied by Muralidharan [13], where the maximum likelihood estimate and the uniformly minimum variance unbiased estimate of it were derived.

Besides the two approaches mentioned above, in this article, however, we investigate the modulated power law process via an objective Bayesian approach. It is a method that has not been used to analyze such a model before. As we all know, choosing an appropriate prior is the main task for Bayesian statisticians. In practice, subjective priors always depend on the experts' belief which is neither easy to quantify in a limited time period nor easy to convince others. Therefore, given little prior information, we prefer to use objective priors to make inference. A reference prior is one of the objective priors and introduced by Bernardo [14]. Other objective priors include Jeffreys prior, probability matching priors, and so on. Different objective priors are defined under different frameworks. In our paper, only reference priors are considered. The basic idea of constructing a reference prior is to construct a density maximizing the missing information about the parameter of interest. Via the objective Bayesian approach, we succeed in doing Bayesian estimation and prediction whose coverage probabilities are close to the nominal level even for small samples. Because of the characteristics of reference priors, they could conveniently be used as standards for scientific communication.

The motivation for us to consider the MPLP from the view of objective Bayes has been mentioned previously. The paper is organized as following. In Section 2, model formulation of the modulated power law process is given, where the Fisher information matrix of the likelihood and Jeffreys prior are calculated. In Section 3, seven reference priors, one of which is also Jeffreys prior, are derived. Some of them perform poorly for their complicated forms and setting undesirable constraints to the parameters. Therefore, only four of the reference priors are taken into consideration for practicality. Bayesian posterior analysis considering the four reference priors are given in Section 4. The propriety of posterior distributions is proved. Procedures for Gibbs sampling are also given in this section. Bayesian prediction of the future failure time is investigated in Section 5. Simulation studies and real data analysis are given in Sections 6 and 7, respectively. The statistical properties of Bayesian estimates are compared, via Monte Carlo simulation, with the maximum likelihood ones in a frequentist context in the simulation studies. Both simulation and application work show the superiority of objective Bayes over classical maximum likelihood method. Therefore, the Bayesian approach using reference priors can be an attractive alternative way to analyze the MPLP.

2. Model formulation

Consider a repairable system experiencing shocks that occur according to the NHPP with power law or Weibull intensity function,

$$h(t) = \lim_{\Delta t \to 0} \frac{P[\text{a shock occurs in } (t, t+\Delta t)]}{\Delta t} = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, \quad t \geq 0, \mathcal{L} \tag{1}$$

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