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Verification and validation of the foredrag coefficient for supersonic and hypersonic flow of air over a cone of fineness ratio 3

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ABSTRACT

The foredrag coefficient resulting from the supersonic and hypersonic flow of air over a cone was calculated numerically using a finite volume approach based on the compressible Euler and Navier-Stokes equations with constant and variable thermophysical properties. No turbulence model was considered. Simulations were carried out for a cone of fineness ratio 3 under the free-stream Mach numbers 2.73, 3.50, 4.00, 5.05 and 6.28 (the Reynolds number, based on cone length, is within 0.45 and 2.85 million). Up to six grids were employed for numerical calculations, with 60 × 60 to 1920 × 1920 volumes. The numerical error was estimated to be less than 0.01% of the numerical solution for all models. Comparisons of the numerical foredrag coefficients of the three models with the experimental data showed that the Navier–Stokes model with variable thermophysical properties agreed better with the experimental foredrag for the entire Mach number interval studied, taking into account the validation standard uncertainty.

1. Introduction

Increased computer power in the past decades has led to a widespread use of numerical methods to solve fluid dynamics problems, followed by an increased interest of the scientific community on the accuracy and reliability of numerical solutions. There are several papers (see, for instance, Ref. [1-4]) and entire books [5–7] addressing this issue.

According to Roache [7], the estimation/quantification of errors/uncertainties in Computational Fluid Dynamics (CFD) are performed through verification and validation. Verification estimates/quantifies the error/uncertainty caused by solving approximately a mathematical model, while validation estimates/quantifies the error/uncertainty caused by the modeling itself. Verification can be divided into code verification and solution verification. Code verification aims to eliminate or, at least, minimize the chance of coding mistakes (bugs), while solution verification aims to estimate/quantify the numerical errors/uncertainties related to the approximations applied to solve the mathematical model.

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Despite the above mentioned progress in CFD and error quantification, based on the authors' experience, verified and tabulated numerical solutions for the flow over basic geometries, such as the conical one, are still not widely found in the open literature. This type of data is especially useful when comparing mathematical models or checking the results of a new software in its early development stage, for instance.

Accordingly, the aim of this work is to numerically determine the foredrag coefficient of the classical conical flow problem [8] and its numerical uncertainty for three mathematical models: (i) Euler equations, (ii) Navier–Stokes equations with constant thermophysical properties (NS-C) and (iii) Navier–Stokes equations with variable thermophysical properties (NS-V). No turbulence model is considered. Additionally, this work aims to estimate the modeling error, and associated uncertainty, of the investigated models by comparing the numerical solutions with the experimental data of Eggers et al. [9].

The calculations are limited to a cone of fineness ratio $f = 3$ (length/base diameter) and to the free-stream Mach numbers 2.73, 3.50, 4.00, 5.05 and 6.28[1]. The free-stream Reynolds number, based on the cone length, vary from 4.5×10^5 to 2.85×10^6. The cone geometry was chosen because the solutions of the Taylor–Maccoll equation [11] (a specialization of the Euler equations for the conical flow) can be obtained so accurately with nowadays computers, that they can be treated as analytical. Moreover, the Mach numbers, Reynolds number and cone aspect ratio were chosen because of the available experimental data of Eggers et al. [9], which are used in the validation.

Verification and validation procedures are applied here based on the recommendations of ASME V&V 20–2009 Standard [12]. During this process, the convergent estimator [13,14] is applied to obtain higher order solutions from the numerical one and some difficulties related to the validation are exposed.

2. Methodology

2.1. Flow simulation

Flow is modeled by the time dependent, axisymmetric, compressible Euler and Navier–Stokes equations [15]. The Euler model is obtained from the Navier–Stokes one by neglecting all terms depending on the viscosity and thermal conductivity. This work focuses on the steady state solution. Time dependence is applied to make the solution algorithm stable.

The thermophysical properties, *i.e.*, viscosity μ, thermal conductivity κ and specific heats at constant pressure c_p and volume c_v, may be considered constant and equal to their free-stream values or they may be functions of the local temperature T. In this study, the fluid (air) is a mixture of Ar, O_2 and N_2 in the mole fractions of $X_1 = 1\%$, $X_2 = 21\%$ and $X_3 = 78\%$, respectively. In order to calculate c_p, μ and κ of the gas mixture, the corresponding thermophysical properties $(c_p)_i$, μ_i and κ_i of each chemical specie i are first calculated according to the interpolation formulas of McBride et al. [16] as

$$(c_p)_i = (R_g)_i\left[A_i + B_iT + C_iT^2 + D_iT^3 + E_iT^4\right], \tag{1}$$

$$\mu_i = \exp\left(A_i' \ln\left(\frac{T}{\mathrm{K}}\right) + \frac{B_i'}{T} + \frac{C_i'}{T^2} + D_i'\right) \cdot 10^{-7}\ \mathrm{Pa\ s}, \tag{2}$$

and

$$\kappa_i = \exp\left(A_i'' \ln\left(\frac{T}{\mathrm{K}}\right) + \frac{B_i''}{T} + \frac{C_i''}{T^2} + D_i''\right) \cdot 10^{-4}\ \mathrm{W\ m^{-1}K^{-1}}, \tag{3}$$

where the gas constant of each specie $(R_g)_i$ and the coefficients A_i to D_i'' are given by McBride et al.

The thermophysical properties for the gas mixture are calculated according to Refs. [15] and [17] as

$$c_p = \sum_{i=1}^{3} X_iM_i(c_p)_i \Big/ \sum_{i=1}^{3} X_iM_i, \tag{4}$$

$$\mu = \sum_{i=1}^{3} \frac{X_i\mu_i}{\sum_{j=1}^{3} X_j\Phi_{ij}^{\mu}}, \tag{5}$$

$$\kappa = \sum_{i=1}^{3} \frac{X_i\kappa_i}{\sum_{j=1}^{3} X_j\Phi_{ij}^{\kappa}}, \tag{6}$$

where

$$\Phi_{ij}^{\psi} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-1/2}\left[1 + \left(\frac{\psi_i}{\psi_j}\right)^{1/2}\left(\frac{M_j}{M_i}\right)^{1/4}\right]^2, \quad \psi \in \{\mu, \kappa\} \tag{7}$$

[1] Some preliminary results were presented in CMAC-SE conference [10].

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