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Bending analysis of thin functionally graded plate under in-plane stiffness variations

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ABSTRACT

The paper developed a new analytical solution for elastic deformation of thin rectangular functionally graded (FG) plates with in-plane stiffness (Young's modulus) variation, which has important applications in various thin-walled structures. Also the problem was solved numerically using the graded finite element method (FEM). The relevant governing equations of elasticity were solved assuming static analysis and power law distribution of the material stiffness. The plate deflections and stresses from the well-known through-the-thickness stiffness variation solution were used to verify the graded finite element method. The analytical solutions for the displacements and stresses were derived for in-plane stiffness variations. The finite element (FE) solutions were obtained both using linear hexahedral solid elements and shell elements with spatially graded stiffness distribution, implemented in the ABAQUS FE software. These solutions were verified against the finite element (FE) solutions, and are in very good agreement for various stiffness gradients. The analytical solution based on CPT was compared with that provided by higher shear deformation theory (HSDT) and graded solid element FE solution.

The results obtained demonstrate that the direction of material stiffness gradient and the nature of its variation have significant effects on the mechanical behavior of FG plate. Moreover, the good agreement found between the exact solution and the numerical simulation demonstrates the effectiveness of graded solid elements in the modeling of FG plate deflection under bending. The types of analytical solutions obtained can be used to obtain deflections and stresses in thin structures with specified stiffness gradients induced by manufacturing processes, such as multi-material 3D printing.

1. Introduction

In examining some natural structures such as the stems of plants, animal bones, mollusk shells, and other biological hard tissues, it can be observed that their geometry or structure changes to accommodate to their physical environment [1,2]. This implies that these structures are highly adapted to all boundary and loading conditions defined by their environment. Functionally Graded Materials (FGMs), which get their inspiration by mimicking many biological structures, are advanced materials with composition and properties change spatially from one surface to another, which are created by specialized

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manufacturing processes [3]. The main advantage of FGMs is the elimination of stress concentration and discontinuity at the interface between matrix and inclusion due to the monotonous variation of volume fraction of the constituents and the resultant mechanical and/or functional properties [4]. Since their introduction by Kawasaki and Watanabe [5] in high temperature metal/ceramic aerospace components, FGMs have become alternative materials widely used in biomechanical, aerospace, civil, automotive and other industrial applications, such as cutting tools, orthopedic, dental implants and gradient alloys for spacecraft panels [6,7].

Although FGMs is heterogeneous material, it could be beneficial to model them as a continuum material with continuous variation of material properties [7]. In order to describe the distribution of material properties, different mathematical formulations can be used, e.g. power-law and exponential law. Mathematical models using a power-law distribution have been widely used in several studies [6–14]. Some researchers have also used exponential functions for defining material property variation [15,16].

One of the primary applications of FGMs can be found in plate structures. Therefore, understanding the mechanical behavior of FG plates is vital for effective design of structures employing FGMs to meet desired performance and safety criteria. The deformation solutions for FG plates have been mainly based on classical plate theory (CPT), which is valid for thin plates, as it does not consider shear deformation. This theory has been implemented by several researchers to study the mechanical behavior of thin FG plates [17–22].

An exact solution for exponentially graded FG plates with simply-supported boundary conditions under a surface load was proposed by Pan [23]. Further, Chi and Chung [21] derived an analytical formulation for three types of distribution functions, namely power-law, sigmoid and exponential functions, based on Classical Plate Theory (CPT) for a rectangular simply-supported FG plate with transverse loading. They evaluated the exact solution with numerical simulation by MARC FE program with 16 layers of different material properties in the thickness direction. Other plate theories which consider shear deformation factors, such as First-order Shear Deformation Theory (FSDT), Third-order Shear Deformation Theory (TSDT) and Higher-order Shear Deformation Theory (HSDT), were employed by many researchers [24–27]. At FSDT, a shear correction factor is needed to satisfy the zero transverse shear stress boundary conditions at the top and bottom of the plate [28]. For avoiding the use of shear correction factors, several Higher-order Shear Deformation Theories (HSDT), such as the Third-order Shear Deformation Theory (TSDT) [29,30], the Sinusoidal Shear Deformation Theory (SSDT) [31,32] and the Hyperbolic Higher Shear Deformation Theory (HHSDT) [33,34], have been proposed. Bourada et al. [35] used hyperbolic shear deformation theory to study static and free vibration of simply supported FG beam with material properties variation through the thickness. They used third unknown displacement functions. The effects due to transverse shear and normal deformations are both included. And their theory accounts for adequate distribution of the transverse shear stresses through the beam thickness and tangential stress-free boundary conditions on the beam boundary surface.

Moreover, Belabed et al. [34] proposed an efficient and simple higher order shear and normal deformation theory for FG plates with material properties variation through the thickness. The mentioned theories are appropriate for thick plates which consider the thickness stretching effect and shear deformation. In this study CPT as a simple plate theory has been used to model thin FG plate with in-plane material stiffness variation which leads to accurate and efficient solutions for thin FG plate [20,21].

During the manufacturing process of FGMs, the reliability requirements of the product should be considered to meet desired or application-specific performance criteria. One approach to produce polymeric FGMs is the use of additive manufacturing (3D printing), which can control local composition and microstructure [36–38]. Furthermore, the gradient distribution (which is a common result of 3D printing) and its relationship with the loading direction will affect the macro stiffness and mechanical behavior of an FG plate.

In the applications reported in the literature, only along-the-thickness variations in material stiffness have been implemented for FGMs. Little attention has been considered to the distribution of material properties along the length, which result in significant in-plane variations in a components material properties. Therefore, the development and implementation of static analysis for thin FG plate as a function of in-plane variation of material parameters is the main contribution of the present paper.

In the context of next generation novel materials and smart structures, such as FG structures, in-plane property variation could play a crucial role in producing structures with controlled mechanical behavior and properties. For example, structures which deflect or deform in user specified ways and produce favorable stress distribution with reduced concentration when loaded. One of the potential applications of the FG plates with in-plane stiffness variation could be for aircraft wings for better adaptation and efficiency of the aircraft by localizing stiffness and density improvement to reduce localized deformations.

For design and analysis of FG plate with an in-plane stiffness variation, simple closed-form and analytical solutions may be useful in rapidly determining deformations (displacements) and stresses in the plate. Therefore, the development of analytical and numerical methodologies for accurate structural response predictions of FG plate with in-plane stiffness variation is the main objective of the present work.

This paper presents novel solutions of the stress and strain fields of thin FG plates with in-plane stiffness variation, which has not been developed till date. Moreover, using the graded solid elements for FE numerical simulation is another novelty of the current work. Then the analytical results compared with FE simulation using graded solid elements. Also, study of relationship between gradient distribution as a common result of 3D printing manufacturing process and the loading direction on mechanical behavior of FG plate is another novelty of this paper.