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Adaptive reliability analysis based on a support vector machine and its application to rock engineering

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ABSTRACT

The response surface method (RSM), a simple and effective approximation technique, is widely used for reliability analysis in civil engineering. However, the traditional RSM needs a considerable number of samples and is computationally intensive and time-consuming for practical engineering problems with many variables. To overcome these problems, this study proposes a new approach that samples experimental points based on the difference between the last two trial design points. This new method constructs the response surface using a support vector machine (SVM); the SVM can build complex, nonlinear relations between random variables and approximate the performance function using fewer experimental points. This approach can reduce the number of experimental points and improve the efficiency and accuracy of reliability analysis. The advantages of the proposed method were verified using four examples involving random variables with different distributions and correlation structures. The results show that this approach can obtain the design point and reliability index with fewer experimental points and better accuracy. The proposed method was also employed to assess the reliability of a numerically modeled tunnel. The results indicate that this new method is applicable to practical, complex engineering problems such as rock engineering problems.

1. Introduction

Response surface methodology (RSM) is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes [1]. It has been widely adopted for reliability analysis in practical geotechnical engineering [2]. Reliability analysis based on RSM provides an effective way to couple standalone numerical programs and reliability analysis methods such as first-order and second-order reliability methods (FORM and SORM). Some researchers have adopted reliability analysis methods to analyze the stability of tunnels [3–10]. The RSM procedure involves selecting sample points, constructing the response surface, and computing the reliability index. The sample points are chosen based on central sampling at and around the mean value/tentative design point. In traditional polynomial-based RSM, a second-order polynomial function is commonly used as a surrogate model. The number of samples required increases in tandem with the order of polynomial used. This can be time-consuming for solving practical geotechnical engineering problems when a high-order polynomial is desired, given the large number of input variables. Therefore, the sampling method is very important for reliability analysis based on RSM, especially for complex engineering problems, such as geotechnical engineering.

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To overcome the above problem, various sampling methods were proposed and applied to the reliability analysis. Bucher and Bourgund [11] proposed an alternative procedure for selecting the sampling points, namely positioning the next central sampling point based on the previous trial design point. Kim and Na [12] proposed a gradient projection method for selecting samples close to the original failure surface. Roussoulya and Petitjeanb [13] proposed a new adaptive Latin Hypercube Sampling where the most significant terms are chosen from statistical criteria and cross-validation method. Zhao and Qiu [14] proposed an efficient response surface method by using the control point instead of the design point considering the compromise between the accuracy and the efficiency. Sun et al. [15] employed Latin hypercube sampling to assess the failure domain and probability as well as reliability index of complex structure. Dey et al. [16,17] applied central composite design and Latin hypercube sampling to the uncertainty analysis of composite structure. Kang et al. [18] applied a moving least squares approximation to RSM that gives higher weights to samples closer to the most probable failure point. Somdatta et al. [19] proposed an improved moving least-squares method to build the response surface in reliability analysis. Dey et al. [20] applied D-optimal design based response surface to the uncertainty analysis of the laminated composite conical shells. Li et al. [2] reviewed and compared the response surface method in slope reliability analysis.

With the development of artificial intelligence, more advanced response surface models in the form of artificial neural networks (ANNs) and support vector machines (SVMs) have the advantage of providing high-order approximations with smaller pools of samples compared to polynomial functions of comparable order. ANN-based response surface is able to avoid the problem of false design points arising from the use polynomial response surface [21]. Deng et al. [22] proposed an ANN-based second-order reliability method and an ANN-based Monte Carlo simulation method. Hosnielhewy et al. [23] proposed an ANN-based response surface method to analyze the reliability of structures. Other researchers have applied ANNs to reliability analysis by combining them with a Monte Carlo simulation, FORM, RSM, or other methods [24–26]. Zhao and Zhao et al. [27,28] have used SVM-based FOSM and SVM-based Monte Carlo simulations to analyze the reliability of a slope and a tunnel, respectively. Dey et al. [17] built the SVR based uncertainty quantification algorithm in conjunction with Latin hypercube sampling. Response surface models in the form of ANN and SVM have the advantage of providing high-order approximations with smaller pools of samples compared to polynomial functions of comparable order [22,29,30].

Response surface method has important applications in the reliability analysis. The sampling method and approximation function are critical to reliability analysis based on RSM, especially for complex engineering problems. However, to the author's knowledge, there are still no definite guidelines to selecting sample points and constructing the response surface. The objective of the present work is for building an effective sampling strategy and improving the efficiency of reliability analysis in practical rock engineering.

Aimed the above objective, a new adaptive sampling strategy is proposed in which sample points are selected based on the difference between the last two tentative design points. A least square support vector machine (LSSVM) [31] is used to approximate the performance function for reliability analysis. Details will be presented in the rest of the paper and the paper is structured as follows. The existing reliability analysis methods are first reviewed. Next, the basic concepts of LSSVM, RSM, and the methods for selecting experiment points are presented. Then, the proposed adaptive reliability analysis approach based on LSSVM is presented and applied to four numerical examples and a tunnel. The results obtained from the new LSSVM method are then compared with results using the classical polynomial-based response surface.

2. The reliability analysis method and response surface

2.1. The Hasofer–Lind index and first-order reliability methods (FORM)

The matrix formulation of the Hasofer–Lind index for correlated normals is [32,33]:

$$\beta = \min_{X\in F}\sqrt{(X-\mu)^T C^{-1}(X-\mu)}, \tag{1}$$

where X is a vector representing the set of random variables x_i, μ is the vector of mean values, C is the covariance matrix, and F is the failure domain. From Eq. (1), the Hasofer–Lind index can be regarded as the minimum distance, in units of directional standard deviations, from the mean-value point of the random variables to the boundary of the limit state surface.

Low and Tang [34,35] presented an alternative interpretation of the Hasofer–Lind index from the perspective of an expanding ellipsoid in the original space of the basic random variables. To obviate the computations of equivalent normal means and equivalent normal standard deviations, in a subsequent paper Low and Tang [36] proposed a new, more efficient algorithm for the FORM using a varying dimensionless number, n_i as:

$$\beta = \min_{X\in F}\sqrt{[n]^T[R]^{-1}[n]}, \tag{2}$$

where $\boldsymbol{n}$ is a column vector of n_i. When the value of n_i is varied during constrained optimization, the corresponding value of x_i, is automatically calculated from:

$$x_i = F^{-1}[\phi(n_i)], \tag{3}$$

where $F^{-1}()$ is the inverse of the original non-normal cumulative distribution function (CDF), $\Phi()$ is the standard normal CDF. In this paper, Eq. (2) is used to compute the reliability index.