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Development of a perfect match system in the improvement of eigenfrequencies of free vibration

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ABSTRACT

It is well known that the standard finite element method (FEM) with overly-stiff effect gives the upper bound solutions of natural frequencies in the free vibration analysis using triangular and tetrahedral elements. In this study, for the first time, this paper aims to improve the prediction of eigenfrequencies through the perfect match between the stiffness and mass matrices. With redistribution of mass in the system, we can tune the balance between stiffness and mass of a discrete model. This can be done by simply shifting the integration points away from the Gaussian locations, while ensuring the mass conservation. A number of numerical examples including 2D and 3D problems have demonstrated that the accuracy of eigenfrequencies is strongly determined by the location of integration points in the mass matrix. With appropriate selection of integration points in the mass matrix, even the exact solution of eigenfrequencies can be obtained in both FEM and smoothed finite element method (SFEM) models.

1. Introduction

In the design of dynamic structure, vibration analysis plays a very important role. The prediction of modal frequencies of a structure is very useful to characterize the resonant vibration in machinery and structures [1]. As the analytical solutions for natural frequencies are only available for some simple geometries [2], the finite element method (FEM) has been widely used in the modal analysis [3–5]. It is well known that the displacement-based fully compatible FEM gives the upper bound solutions of eigenfrequencies [6]. This is because the stiffness in the FEM model is stiffer than the exact model [3,4,7]. In order to improve the accuracy of FEM solutions, many efforts have been made. Marangoni et al. [8] developed upper and lower bounds to the natural frequencies of vibration of clamped rectangular orthotropic pates. Wiberg et al. [9] presented the local and global updating techniques to improve FEM solution of the generalized eigenvalue problem in free vibration analysis. An error estimate and an adaptive procedure for generalized linear eigenvalue and eigenvector computation within a framework of the hierarchical finite element method were proposed by Friberg et al. [10]. Mackie [11] modified the standard FEM using the results of dispersion analysis of numerical approximation to improve the accuracy of eignefrequencies of high mode shape.

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In order to soften the overly-stiff FEM model, Liu and co-workers have developed a G space theory [12] and weakened weak form, leading to the foundation of a family of smoothed finite element method (SFEM) [12–15]. By employing the node-based smoothed technique, node-based smoothed finite element method (NS-FEM) is developed with overly-soft property, which always gives the lower bound solutions of eignefrequencies [16,17]. However, NS-FEM can produce the so-called spurious modes with excessive node-based smoothing technique [16,17]. Due to this reason, an edge-based smoothed finite element method (ES-FEM) is developed in order to achieve a close to exact stiffness [7,18–21]. The ES-FEM gives much more accurate solutions in the prediction of eigenfrequencies compared with overly-soft NS-FEM and overly-stiff FEM [17].

The above efforts to reduce the softening effect of discretized mode are all focused on the modification of stiffness. Recently, Guddati [22,23] proposed a new modified integration rules in the computation of the mass and stiffness for acoustic problems with quadrilateral mesh. Following this, He et al. [24] proposed a mass redistributed finite element method (MR-FEM) using triangular elements in the analysis of acoustic problem in order to balance the discretized model between acoustic mass and stiffness. The implementation of MR-FEM is modified by shifting the integration points away from the usual Gaussian locations in the mass matrix [25].

Motivated by the excellent features of MR-FEM in the acoustic field, for the first time this work analyses the effect of mass redistribution in the prediction of eigenfrequencies for different discretized models including FEM and SFEM. Due to complicated geometry in general engineering problems, the high quality elements such as quadrilateral and brick elements are very difficult (or impossible) to generate. Therefore, the triangular or tetrahedral elements are considered particularly popular in industry. In this work, our study shows that the perfect match of discretized model between elastic mass and stiffness can be achieved with the modification of integration point in the mass matrix, which can improve the computational efficiency of eigenfrequencies in both FEM and SFEM models significantly. In addition, the relationship between the eigenfrequency and integration point t of mass matrix has been derived in this work. More importantly, the numerical results have indicated that the upper and lower bound solutions from NS-FEM and FEM models are furthered improved using coarse mesh. This significant finding is very useful to design the mechanical part or structure to ensure that the resonance leading large oscillation and damage never appears.

The paper is organized as follows: Section 2 briefly describes the balanced systems in the discretized model. The formulation of MR-FEM in the computation of eigenfrequencies is shown in Section 3. Numerical examples including 2D and 3D are presented in Section 4 to investigate the effect of integration point of mass matrix on eigenfrequencies using FEM, NS-FEM and ES-FEM with combination of MR-FEM. Finally the conclusions from the numerical results are made in Section 5.

2. The balanced system of discretized model

In the prediction of eigenfrequencies of elasticity systems, the global stiffness matrix and mass matrix are system matrices, and the balance between them is critical to yield accurate results. In the standard Galerkin FEM system, the weak form of FEM leads to the differences in "stiffness" between the numerical model and the exact system, and an imbalance between the stiffness matrix and mass matrix emerges. Hence, the eigenfrequencies in the FEM system are always larger than the real ones [6].

Consider the general elasticity problems without boundary conditions, as the stiffness and mass are positive definite matrices using discretized methods, there always exist an eigenvector $\boldsymbol{\phi}$

$$\boldsymbol{\phi}^T\mathbf{K}\boldsymbol{\phi} = \omega^2\boldsymbol{\phi}^T\mathbf{M}\boldsymbol{\phi}, \tag{1}$$

where $\boldsymbol{\phi}^T\mathbf{K}\,\boldsymbol{\phi}$ and $\boldsymbol{\phi}^T\,\mathbf{M}\boldsymbol{\phi}$ are diagonal matrices, and $\boldsymbol{\phi}$ is the modal matrix.

$$\boldsymbol{\phi}^T\mathbf{K}\boldsymbol{\phi} = \begin{bmatrix} \lambda_{k1} & & & \\ & \lambda_{k2} & & \\ & & \ddots & \\ & & & \lambda_{kn} \end{bmatrix}, \quad \boldsymbol{\phi}^T\mathbf{M}\boldsymbol{\phi} = \begin{bmatrix} \lambda_{m1} & & & \\ & \lambda_{m2} & & \\ & & \ddots & \\ & & & \lambda_{mn} \end{bmatrix}, \tag{2}$$

where $\lambda_{k1} \sim \lambda_{kn}$ are the eigenvalues of stiffness matrix $\mathbf{K}$, $\lambda_{m1} \sim \lambda_{mn}$ are the eigenvalues of mass matrix, and the eigenvalues of elasticity are obtained as follows:

$$\omega^2 I = \frac{\boldsymbol{\phi}^T\mathbf{K}\boldsymbol{\phi}}{\boldsymbol{\phi}^T\mathbf{M}\boldsymbol{\phi}} = \begin{bmatrix} \frac{\lambda_{k1}}{\lambda_{m1}} & & & \\ & \frac{\lambda_{k2}}{\lambda_{m2}} & & \\ & & \ddots & \\ & & & \frac{\lambda_{kn}}{\lambda_{mn}} \end{bmatrix}. \tag{3}$$

From Eq. (3), it can be found that the eigenvalues are determined by the stiffness and mass matrices. In order to improve the prediction of eigenfrequencies of free vibrations, one way is to soften the stiffness of the FEM. With a generalized gradient smoothing technique, smoothed finite element method (SFEM) has demonstrated that the stiffness of the discretized model is reduced compared with the model of FEM. In this work, the mass matrix is modified using MR-FEM to improve

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