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Stability of hydrostatic-pressured FGM thick rings with material nonlinearity

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ABSTRACT

Buckling behaviors of elastoplastic ceramic/metallic functionally graded material (FGM) rings are investigated by using the first order shear deformation theory. The hydrostatic-pressured rings are assumed to be in both the plane-stress case and the plane-strain case, which lead respectively to a uniaxial and a biaxial elastoplastic stress states in prebuckling stage. A uniform strain hypothesis helps to deal with the elastoplastic stress states. By introducing in the graded material properties, the constitutive model of FGMs is formulated under the framework of J_2 deformation theory. By considering the kinetic relations of von-Kárman type and employing the principle of virtual displacement, the equilibrium equations and the buckling governing equations of FGM circular rings are formulated, and the analytical solution of the anisotropic rings is obtained. Finally, the elastoplastic buckling problem is numerically solved through a semi-analytical method, which is proposed to seek the real circumferential strain of FGM rings at the buckling point and determinate the elastoplastic buckling critical hydrostatic pressure. The effects of the inhomogeneous and geometrical parameters on the buckling critical load and the position of the elastoplastic interface are discussed. Results show that, in both the plane-stress and the plane-strain cases, the elastoplastic critical loads are generally lower than their elastic counterparts due to material flow, and the plane-strain critical load is generally larger than the plane-stress one. The elastoplastic critical load does not always decrease monotonously with the increase of the inhomogeneous parameters, which is quite different from their elastic counterparts.

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1. Introductions

As fundamental component or simplified model of pressure-loaded hole cylindrical structures, such as undersea oil transportation pipelines, submarines, and etc., researches in mechanical performances of circular rings have received extensive interest, especially their buckling behaviors, which are of great concern to submersible structural designers.

Early researches on buckling of homogeneous rings were Timoshenko and Gere [1] and Smith and Simitses [2], which gave the analytical solutions for linear buckling of elastic thin rings under in-plane loads. Investigations on postbuckling behaviors might initiate by Flaherty et al. [3], whose work focused on the large deformation analyses with opposite sides of rings in contact, and the subsequent postbuckling theories were formulated under the inextensional or extensional assumption of the rings. Kyriakides and Babcock [4] presented elastoplastic postbuckling analyses for imperfect inextensional rings

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and found that the inelastic nature of the material creates a distinct limit critical load, beyond which the response path is unstable. Nevertheless, for elastic circular rings, the postbuckling path is stable, as a prime result of Fu and Waas [5], where thickness effects were discussed for extensional thick ring by using an approximate first order shear deformation theory. Kim and Chaudhuri [6] employed a fully nonlinear finite-element method to study the postbuckling issue of moderately thick imperfect rings under external pressure, the transverse normal and shear strain effect were examined, and the error of von-Kárman nonlinear theory was assessed. Wu et al. [7] applied the combinations of an improved harmonic balance method and Newton's technique to obtain the approximate analytical solutions for the nonlinear differential equations of the post-buckling response of the inextensional ring.

On buckling behaviors of circular rings made of composite materials, Jones and Morgan [8] obtained analytical solutions to predict the buckling critical pressures of thin cross-ply laminated rings and cylindrical shells. Kim and Chaudhuri [9] investigated buckling behaviors of perfect thick plane-strain cross-ply rings by using finite element method. Asemi and Kiani [10] studied postbuckling behaviors of polar orthotropic linearly elastic rings in the plane-stress case, and subjected to external pressure. In their work, the Newton–Raphson iterative technique was applied to calculate the post-buckling response of the ring up to the collapse point.

Recent emergence of ceramic/metallic FGMs has raised enormous research concerns about buckling behaviors of FGM structures. Kadoli and Ganesan [11] discussed thermal buckling behaviors of clamped FGM cylindrical shells. Bagherizadeh et al. [12] investigated buckling issue of FGM cylindrical shells embedded in elastic medium and subjected to combined axial and radial loads. Wu et al. [13] presented linear buckling analysis for simply-supported, multilayered FGM circular hollow cylinders under combined axial compression and external pressure. Yaghoobi and Fereidoon [14] investigated mechanical response and thermal buckling issues of FGM plates resting on elastic foundation. Shen [15] investigated postbuckling behaviors of FGM cylindrical shells and plates under thermal loads by using boundary layer theory. Woo et al. [16] focused on postbuckling behaviors of FGM plates induced by thermal loads. Tung [17] presented an analytical approach to investigate the effects of tangential edge constraints on postbuckling behaviors of FGM flat and cylindrical panels resting on elastic foundations and subjected to thermo-mechanical loads. For FGM ring structures, literature is still limited. Kerdegarbakhsh et al. [18] dealt with elastic buckling and postbuckling problems for FGM rings by using the first order shear deformation theory. However, been not taking into account the material nonlinearity, the buckling behaviors of thick FGM rings predicted by Kerdegarbakhsh may be different enormously from the real ones.

Currently, little research interest has been attracted on buckling of FGM rings, especially for elastoplastic buckling. As is generally known, the submarine structures must be sufficiently stable to sustain large hydrostatic pressure. One of the most effective ways is to increase the thickness dimension of their components. However, thicker geometry tends to induce larger stress and lead to material flow. Therefore, it is essential to consider the material elastoplasticity for thick components.

In this paper, elastoplastic buckling analysis of FGM circular rings is presented. The rings are assumed to be in either the plane-stress case or the plane-strain case. As an extension of our previous work [19], in which elastoplastic buckling response of FGM cylindrical shells under external pressure was investigated by using the classical shell theory, this paper deals with elastoplastic buckling problem of thick FGM rings using the first order deformation theory. The constitutive model is formulated under the framework of J_2 deformation theory, and both the uniaxial and the biaxial elastoplastic stress states in prebuckling stage are analyzed based on the uniform circumferential strain hypothesis. A semi-analytical method helps to seek the real circumferential strain of FGM rings and then determinate the elastoplastic buckling load .

2. Formulations

2.1. General descriptions

The FGM ring with the thickness h and the mean radius R is plotted in Fig. 1. r and θ denote the radial and the circumferential coordinate axes. z axis measures from the mid-plane along the radial direction and $z=r-R$. x axis is in the normal direction of the ring's plane.

2.2. Material properties and the constitutive model

The constituent distribution of FGMs considered herein follows the power exponent rule [11], given as:

$$f_\mathrm{c} = (0.5 - z/h)^k,\ f_\mathrm{m} = 1 - f_\mathrm{c}, \tag{1}$$

where f is the volume fraction, the subscripts c, m denote the ceramic and metallic phases respectively. k is the inhomogeneous parameter governing the constituent distribution.

The elastoplastic material properties of FGMs can be formulated by the TTO model, initially proposed for metallic alloy by Tamura et al. [20] and Nakamura et al. [21] It can be given in the following form:

$$\begin{aligned} E &= \left[E_\mathrm{m} f_\mathrm{m} (\tilde{q} + E_\mathrm{c})(\tilde{q} + E_\mathrm{m})^{-1} + E_\mathrm{c} f_\mathrm{c}\right]\left[f_\mathrm{m}(\tilde{q} + E_\mathrm{c})(\tilde{q} + E_\mathrm{m})^{-1} + f_\mathrm{c}\right]^{-1} \\ \nu &= \nu_\mathrm{m} f_\mathrm{m} + \nu_\mathrm{c} f_\mathrm{c} \\ \sigma_\mathrm{Y} &= \sigma_\mathrm{Ym}\left\{f_\mathrm{m} + f_\mathrm{c} E_\mathrm{c}(\tilde{q} + E_\mathrm{m})[(\tilde{q} + E_\mathrm{c})E_\mathrm{m}]^{-1}\right\} \end{aligned}$$

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