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A hybrid conjugate finite-step length method for robust and efficient reliability analysis

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ABSTRACT

The robustness and efficiency of the first-order reliability method (FORM) are the important issues in the structural reliability analysis. In this paper, a hybrid conjugate search direction with finite-step length is proposed to improve the efficiency and robustness of FORM, namely hybrid conjugate finite-step length (CFSL-H). The conjugate scalar factor in CFSL-H is adaptively updated using two conjugate methods with a dynamic participation factor. The accuracy, efficiency and robustness of the CFSL-H are illustrated through the nonlinear explicit and structural implicit limit state functions with normal and non-normal random variables. The results illustrated that the proposed CFSL-H algorithm is more robust, efficient and accurate than the modified existing FORM algorithms for complex structural problems.

1. Introduction

The engineering problems inherently involved various uncertainties in material properties, geometric and external loads [1,2]. Structural reliability analysis and sensitivity analysis are widely used to evaluate the failure probability using probabilistic model [1–4]. The failure probability is computed by solving a multi-dimensional integral based on limit state functions (LSF) as follows [2,5,6]:

$$P_f = \int_{G(\boldsymbol{X})\leq 0} \ldots \int f_{\boldsymbol{X}}(x_1, \ldots, x_n) dx_1 \ldots dx_n, \tag{1}$$

where, $G(\boldsymbol{X})$ is the LSF, which separates the design domain into failure ($G(\boldsymbol{X}) < 0$) and safe ($G(\boldsymbol{X}) > 0$) regions with respect to basic random variables $\boldsymbol{X}=(x_1,\ x_2,\ \ldots,\ x_n)^T$. $f_{\boldsymbol{X}}$ is the joint probability density function (PDF) of X. Generally, Monte Carlo Simulation (MCS) [7,8], impotence sampling (IS) [9,10], first-order reliability method (FORM) [5,11,12] and second-order reliability method (SORM) [13–15] are implemented to estimate the failure probability. The MCS and IS are computationally inefficient approaches to evaluate the failure probability of structural problems with implicit LSFs and low failure probabilities. The FORM is widely used in reliability analysis due to its accuracy and efficiency. Hasofer and Lind (1974) [11] proposed an iterative formula of FORM that Rackwitz and Fiessler [16] extended the Hasofer and Lind formula based on distribution of basic random variables (called as HL-RF method). After that, Liu and Kiureghian [17] studied several optimization schemes indulging the gradient projection, augmented Lagrangian, and the sequential quadratic programming in reliability analysis-based FORM.

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Nomenclature

P_f	failure probability
$f_{\boldsymbol{X}}$	joint probability density function
$G(\boldsymbol{U})$	limit state function in standard normal space
$G(\boldsymbol{X})$	limit state function in X- space
$\boldsymbol{d}_k$	conjugate gradient vector
$\boldsymbol{U}*$	most probable point (MPP) in U-space
$\boldsymbol{X}$	basic random variables in *X*-space
$\boldsymbol{\alpha}_k^{c\lambda}$	normalized conjugate direction vector (sensitivity vector)
β	reliability index
λ	control factor, step size or finite-step length
η	participation factor
CFLS-H	conjugate finite-step length with hybrid search direction
FORM	first-order reliability method
FSL	finite-step-length method
HL-RF	Hasofer–Lind and Rackwitz–Fiessler method
LSF	limit state functions
MCS	Monte Carlo Simulation
Relaxed HL-RF	RHL-RF
SORM	second-order reliability method
STM	stability transformation method
IS	importance sampling

The HL-RF scheme may produce unstable results such as periodic and chaotic solutions in highly nonlinear problems [5,18,19]. Liu and Kiureghian [17] introduced merit function to improve the robustness of HL-RF method. Santosh et al. [20] improved the HL-RF scheme by selecting a step size based on Armijo rule. Yang [19] proposed the stability transformation method (STM) with chaos feedback control to achieve the stabilization of FORM. Gong and Yi [3] proposed a FORM formula using finite-step length (FSL). The FSL and STM are more robust than the HL-RF method, but number of iterations to achieve stable results may be increased, when the step length selected too small in the STM. Recently, Keshtegar and Miri [5,18] improved the HL-RF method based on conjugate gradient optimization method [5] and the relaxed approach (i.e., relaxed HL-RF) [18]. The Wolfe conditions, which are criterion to determine a step size in order to satisfy the sufficient descent [5,21] was used in FORM-based conjugate gradient optimization [5] and inverse reliability method [21]. The relaxed HL-RF (RHL-RF) was developed based on a dynamic step size, which was computed using second order fitting between 0 and 1 [18] or sufficient descent conditions between 0 and 2 [21]. The FORM formula proposed in Refs. [5,18,21] is robust but more the computational formulations to compute the step size. The modified algorithms of FORM are more robust than the HL-RF for highly nonlinear performance functions, but are inefficient for linear and moderately nonlinear reliability problems [22–25]. The Armijo rule [24–25] and sufficient descent condition [23–26] were used to determine a step size based on an inner loop to control the instability of FORM in chaotic conjugate chaos control [22] and an adaptive formula in chaotic conjugate STM [23], limited conjugate search direction [24] and the conjugate HL-RF [25] methods. Consequently, the efficiency and robustness of FORM are the important issues to select an iterative formula in reliability analysis.

In this paper, a hybrid conjugate search direction with finite-step length is proposed based on Conjugate Descent (CD) [27] and Rivaie, Mustafa, Ismail and Leong (RMIL) [28] conjugate optimization methods to improve the efficiency and robustness of FORM in the reliability analysis. A novel finite-step length is developed without line search rules (e.g. Wolfe conditions) and merit function, which is named the conjugate finite-step length (CFSL). This work consists of six sections as follows: a brief review of HL-FR [8], the STM [16], the FSL [3], and RHL-RF [12] is given in Section 2. Following that, Section 3 presents a conjugate finite-step length with hybrid search direction (CFLS-H). In Section 4, a simple step length is introduced for the CFSL-H algorithm. Section 5 presents six numerical examples to illustrate the robustness and efficiency of the CFSL-H method. The last section provides the conclusions.

2. First-order reliability method (FORM)

In FORM, the structural failure probability is estimated by linearizing the limit state function on the failure surface using the reliability index (β) as $P_f \approx \Phi(-\beta)$. The reliability index can be determined by following optimization problem [18]:

$$\begin{array}{ll} find & \boldsymbol{U}* \\ Minimize & \beta = (\boldsymbol{U}^T\boldsymbol{U})^{1/2} \\ subjected & to\ G(\boldsymbol{U}) = 0, \end{array} \tag{2}$$

where, $\boldsymbol{U}^*$ most probable point (MPP) and $G(\boldsymbol{U})$ is the LSF in standard normal space. The basic process of FORM is to search the MPP, i.e.,$\beta = \|\boldsymbol{U}^*\|$ [2]. The MPP can be searched based on an iterative process by following methods: