Applied Mathematical Modelling 45 (2017) 336–364

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Static and dynamic analyses of isogeometric curvilinearly stiffened plates

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ARTICLE INFO

Article history:
Received 18 April 2016
Revised 9 November 2016
Accepted 31 December 2016
Available online 6 January 2017

Keywords:
IGA
Curvilinearly stiffened plates
Static deformation
Free vibration
Large deformation
Large amplitude vibration

ABSTRACT

The isogeometric analysis (IGA) is a new approach which builds a seamless connection between Computer Aided Design (CAD) and Computer Aided Engineering (CAE). This approach which uses the B-Splines or the Non-Uniform Rational B-Splines (NURBS) as a geometric representation of the object is a discretization technology for numerical analysis. The IGA has advantages of capturing exact geometry and making the flexibility of refinement, which results in higher calculation accuracy. To study the static and dynamic characteristics of curvilinearly stiffened plates, the NURBS based isogeometric analysis approach is developed in this paper. We use this approach to analyze the static deformation, the free vibration and the vibration behavior in the presence of in-plane loads of curvilinearly stiffened plates. Furthermore, the large deformation and the large amplitude vibration of the curvilinearly stiffened plates are also studied based on the von Karman's large deformation theory. One of the superiorities of the present method in the analysis of the stiffened plates is that the element number is much less than commercial finite element software, whereas another advantage is that the mesh refinement process is much more convenient compared with traditional finite element method (FEM). Some numerical examples are shown to validate the correctness and superiority of the present method by comparing with the results from commercial software and finite element analysis.

1. Introduction

Curvilinearly stiffened structures have been widely used as unitized structures for decades. These structures can ensure the reliability and durability as well as save the structure material and reduce the weight, which improve the utilization efficiency and economy. Curvilinear stiffeners in the aircraft design have been used to enhance design space for aircraft wings and reduce the structure weight [1,2]. Besides, the curvilinearly stiffened structures have been used as aircraft fuselage, airframe structure and so on [3].

During the last several decades, an increasing number of researches on stiffened plates have been conducted. Rossow and Ibrahimkhail [4] used FEM to analyze the static stiffened plates in which the displacement of ribs is represented by that of middle plane and the energy equation of the structure is established. What's more, the analysis accuracy of several approximate polynomial functions is investigated. Srinivasan and Thiruvenkatachari [5] analyzed the static deformation of stiffened plates and presented a lot of numerical experiments. Oleary and Harari [6] adopted FEM to study the static behavior of

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http://dx.doi.org/10.1016/j.apm.2016.12.035

stiffened plates. Yukio and co-workers [7] used incremental method to investigate the stiffened plates. Bedair presented some researches on stiffened plate structures, e.g. (1) the influence of the stiffeners' distribution on structure's stability under the compression or bending load [8]; (2) the estimation of the ultimate strength of the cross-ply stiffened steel plate [9]; (3) the mechanical properties of the stiffened plate with a plurality of stiffeners under uniform pressure [10]; and (4) the summary of some analysis and design method for stiffened plate and shell structures [11]. Olson and Hazell [12] and Olson [13] studied the vibration problem of stiffened plates and summarized some early research work. Rao and co-workers [14] researched the vibration of cantilever stiffened plates. Bedair and Troitsky [15] discussed the characteristic frequency of simply supported stiffened plates, and analyzed on the vibration of stiffened plates with concentric or eccentric stiffeners. Liu et al. [16] worked on the sensitivity analysis of the characteristic frequency of stiffened plates. Saadatpour et al. [17] did static and vibration analyses on stiffened plates in which some numerical experiments were used to study the influence of the parameters on the mechanical properties of the stiffened plate.

In addition, geometric nonlinearity and material nonlinearity of stiffened plates have also been studied. Turvey and his co-workers [18–21] carried out a lot of researches on circular stiffened plates and square stiffened plates such as the elastic deformation analysis and the large deformation analysis with the geometric nonlinearity. Kolli and Chandrashekhara [22,23] analyzed the mechanical properties of composite stiffened panels under shearing load and studied the displacement, vibration and buckling properties based on geometrically nonlinear large deformation theory. Based on the large deflection theory of plate, Rao et al. [24] investigated the large amplitude vibration and considered the influence of amplitude coefficient on the natural frequency of stiffened plates. The results showed that the nonlinear term of the strain made the natural frequency be larger, and the natural frequency increased as soon as the amplitude coefficient increased. Sheikh and Mukhopadhyay [25] used FEM to study the large amplitude vibration of stiffened plates and the effect of the laying way and eccentricity on the frequency of structures. Mitra et al. [26] researched the large amplitude vibration problem of stiffened plates with free boundary.

Analytical solutions to study the above problems are useful but often difficult to be found unless some geometries and boundary conditions are relatively simple. Numerical methods have been used to find the approximate solutions of these problems. In the development of advanced computational methodologies, Hughes et al. [27] have proposed a NURBS-based isogeometric analysis (IGA) to integrate CAD into structural analysis. In contrast to FEM, isogeometric approach utilizes NURBS that are common in CAD. The IGA which can easily achieve the smoothness with arbitrary continuity order shows us computational advantages over standard FEM. Moreover, exact representation of shapes even at the coarsest level of discretization, simple and systematic refinement strategy, and more accurate modeling of complex geometries make IGA become a very robust approach. In recent years, IGA has been used in many areas such as turbulence [28], fluid–structure interaction [29], incompressibility [30], structural analysis [31,32], shells [33] and phase-field analysis [34]. For structural mechanics, IGA has been extensively studied for structural vibrations [32], the composite Reissner–Mindlin plates [35], Kirchhoff–Love shells [36,37], the large deformation with rotation-free [38] and structural shape optimization [39]. In this paper, a NURBS-based isogeometric analysis formulation is presented to study static and vibration problems of curvilinearly stiffened plates.

The paper is organized as follows: In Section 2, a brief introduction of the B-spline and NURBS basis functions is considered. After that, formulations of isogeometric analysis method are presented. In Section 3, the model of curvilinearly stiffened plates is first set up. Then, static analysis, free vibration analysis and vibration analysis under the in-plane load, large deformation analysis and large amplitude vibration analysis of the curlinearly stiffened plates are carried out. Section 4 is then devoted to numerical tests which show the performance of the proposed method. In Section 5, we close this paper with some conclusions.

2. Isogeometric analysis method

The fundamental idea of IGA is to use the same basis functions which model precise geometries as those of numerical solution space. The calculation unit (also called as element) is constructed by a knot span in the original knot vector. The shape function adopted is NURBS basis function. The control parameters of the corresponding fields on the control points are considered in the implementation of IGA. In this section, we first begin with NURBS's introduction, then present some basic formulations of IGA.

2.1. Brief introduction of NURBS

NURBS basis function is a linear combination of weighted B-spline basis functions, so we first discuss B-splines, and then NURBS.

2.1.1. B-spline basis functions

A knot vector is a sequence in an ascending order of parameter values, written as $\{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$, where ξ_i is the ith knot, n is the number of basis function and p is the polynomial order. The knot vector divides the parametric space into intervals usually referred to as knot spans. The associated B-spline basis functions are defined recursively starting with