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Stability analysis of an open cracked rotor with the anisotropic rotational damping in rotating operation

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ABSTRACT

The damping effects with the distinction of stationary damping and the anisotropic rotating damping on the dynamic stability of the rotating rotor with an open crack on the surface of the shaft is studied. The motion equations of the cracked rotor system are formed by Lagranges principal. Different from previous studies, the anisotropic system with the multi periodical varied coefficients is simplified by the moving frame method such that the stability analysis based on the root locus method can be applied. The corresponding Campbell diagram, decay rate plot and roots locus plot are derived to prove the destabilizing influence of both the rotational damping and the varied anisotropy ratio of the rotating damping. The effects of anisotropy of stiffness on the decisions of the critical range are also presented. The result with theoretical precision would not only generally provide practical applicability to crack detection and instability control of the heavy loading turbo-machinery system, but also give the suggestion that, the increased proportion and the aggravated anisotropy of the rotational damping due to the crack of the fatigue rotor should been taken into consideration on the modeling of cracked rotor system.

1. Introduction

Fatigue cracks of rotating machinery shaft have aroused the attentions over years for the sake of crack detection and instability control to avoid catastrophic engineering accidents [1,2]. The anisotropy of the stiffness varied in an elliptic way with the same period of the rotation speed is considered as the basic property of the modeling of the rotating cracked rotor.

The motion equations formed in inertial reference frame are periodical time-varied linear differential equations, a sort of parametrically excited system on which the theoretical stability analysis with rigorous mathematical logic is very hard to be carried out, are normally studied by means of numerical approach in the existing literature. The numerical studies are mainly in two ways. One is the spectrum analysis which aims at the frequency components analysis of both phase and amplitude to provide the indicators of the crack [3–6]. The other way is the numerical stability analysis of the dynamic motion and tries to give the options for instability control [7–14].

The stability approach base on Floquet's theory is applied not only on the bifurcations [7,10,14], but also act as a qualitative methodology for stability estimation of the nonlinear motion [10–14]. Bolotin's method is a quick evaluation method modified from Floquet's method [8,9]. By searching the boundaries of instability zones directly, the improved stability

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Nomenclature

$0-xyz$	stationary coordinates
c	center of disc
$0xy$	stationary cross section of cracked shaft
$0z$	axial direction of rotating shaft
$0\xi\eta$	rotating coordinates on the cross section of cracked shaft
0ξ	crack direction
m	mass of disc
a	crack depth
ϕ	unbalanced oriented angle measured from the crack direction
Ω	the rotation speed of rotor
D	diameter of the shaft
$c_{r\xi}$, $c_{r\eta}$	rotational damping coefficient
c_n	rotational damping coefficient
β	the anisotropy parameter for rotational damping
α	the anisotropy parameter for stiffness
γ	the dimensionless ratio of rotational to stationery damping
k_0	the stiffness of the uncracked shaft
k_ξ	the stiffness of the cracked rotor in ξ direction
k_η	the stiffness of the cracked rotor in η direction
C_ξ	the additional compliance of the cracked shaft in ξ direction
C_η	the additional compliance of the cracked shaft in η direction
$C_{\xi\eta}$	coupling compliance coefficients
C_0	the compliance of the uncracked shaft
ε	mass eccentricity of disc
$s=\sigma+i\tilde{\omega}$	the complex parameters of Laplace transformation in the rotating coordinates
σ	the attenuation index of amplitude of the vibration
$\tilde{\omega}$	the whirl speed in the rotating frame
ω	the whirl speed in xoy-plane
Ω	the rotation speed
$\theta=\Omega t+\phi$	the position angle of the cracked rotor
$\Omega'=\frac{\Omega}{\omega_0},\ (\omega_0=\sqrt{\frac{k_\xi+k_\eta}{2m}})$	the dimensionless rotation speed
$s'=\frac{s}{\omega_0}=\sigma'+i\tilde{\omega}'$	the dimensionless complex parameters of Laplace transformation in the rotating coordinates
$\sigma'=\frac{\sigma}{\omega_0}$	the dimensionless attenuation index of amplitude of the vibration
$\tilde{\omega}'=\frac{\tilde{\omega}}{\omega_0}=\omega'-\Omega'$	the dimensionless whirl speed in the rotating frame
$\omega'=\frac{\omega}{\omega_0}$	the dimensionless the whirl speed in xoy-plane

analysis becomes more efficient [15,16]. Although the numerical approach of Floquet transition matrix, which is considered as a common way on the studying on the stability of cracked rotor system, has a certain engineering practical significance, the approximation results based on the digital simulation of the transient response can hardly provides the theoretical precision and reliability.

In the existing literature about rotating dynamics, the effect of viscous damping is generally believed to be an important stabilizing factor. Guo et al. [10], Ricci and Pennacchi [11] and Huang et al. [12] demonstrated the stability effects of damping by means of numerical approach based on Floquet's theory. Applying Floquet's method and the generalized Bolotin's method, Kulesza and Sawicki [15] and Ecker [16] investigated the anti resonant effect of the parametric excited damping on the digital simulation of the transient response. While Dohnal et al. [17–19] committed to derive the anti resonant effect from the experimental data.

Genta and Brusa [20] found a interesting damping effect on rotating dynamics that, different from the stationary damping, which is usually designed on the stationary parts to satisfy the required stability in the rotating machine operating range, the rotational damping, usually directly connected with the rotor, is often an unstable factor especially in the super critical region though it may attenuate the oscillations at the sub-critical range.

Due to the fatigue crack on the surface of the rotor, the portion of rotational damping is aroused significantly and the anisotropy property is taken place not only on the stiffness but also on the rotating damping. The variation of the stiffness of the cracked rotor is well studied [2–6,21,22]. The rigid finite element method, by which the stiffness variation is derived by the flexibility matrix for finite element of the shaft with an open crack based on the fracture mechanics, is first proposed by Dimarogonas and Paipetis [21]. Utilizing the sign of stress intensity factors and the energy release rate, an improved