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Modelling functionally graded materials in heat transfer and thermal stress analysis by means of graded finite elements

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ABSTRACT

For better understanding of the behaviour of functionally graded materials (FGM) in high temperature environment, a reliable and efficient numerical tool is required for predictions of heat transfer behaviour and thermally-induced stresses in them. This study presents a finite element formulation of a coupled thermo-mechanical problem in functionally graded metal/ceramic plates. The theoretical framework considers the finite element method (FEM) which is applied to the development of a functionally graded two-dimensional plane strain finite element. The plane strain graded finite element is incorporated within the ABAQUS™ code via the combination of user-defined subroutines. The subroutines enable us to program graded mechanical and thermal properties of the FGM as continuous position-dependent functions and, then, to sample them directly at the Gauss integration points of the element. The performance of the developed graded finite element is verified by comparisons with results known in the literature and with calculated using conventional homogeneous elements in a layered model. The solutions of thermo-mechanical problems of functionally graded plates referring to pure mechanical and thermal tasks, and uncoupled and coupled analyses of thermoelasticity are carried out and discussed in the paper.

1. Introduction

Functionally graded materials (FGMs), originally developed as thermal barrier coatings for aerospace structures and fusion reactors have nowadays received a wide spread as structural components in transportation, energy, electronics and biomedical engineering for the general use in the high temperature environment [1]. One of typical FGMs are functionally graded metal/ceramic composites which offer optimal thermo-mechanical characteristics, also they allow one to avoid interfacial delamination usual for layered compositions of a ceramic coating and a metal substrate [2]. The behaviour of FGMs including heat transfer problems and thermal stress analyses in metal/ceramic FGMs have been extensively studied in the past decade, e.g. [3,4] among many others. However, to respond to demands of new structures and technologies connected with further increasing the in-service temperature, the thermal behaviour and efficiency of FGMs should be better

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understood. The primary interest of this, in essence, is the examination of the thermally-induced stresses that have a critical relevance to fracture mechanisms in FGM structural elements [5–7]. Therefore, it is of huge importance to have reliable and efficient computational models for evaluating complex thermo-mechanical response of FGM structures at a design stage.

A comprehensive review of the principal theoretical developments in functionally graded materials with an emphasis on studies on heat transfer issues, stress, stability, dynamic and fracture analyses, testing, manufacturing, design and applications has been reported in [8]. From this review, it is obvious that heat transfer and thermal stress analysis of FGM is analytically complicated to be performed, as a result the closed-form solution of these problems is possible only for few cases with particular types of thermal loads, boundary conditions and inhomogeneity. Although it is worth mentioning that direct approaches based on the original idea of Cosserat possess a high potential to analytical predictions of thermo-mechanical response in FGMs as shown in [9], where a functionally graded plate was successfully analysed. Nevertheless, in general, closed-form solutions are difficult, if not impossible, to be found for FGM plates of finite dimensions and where the material properties vary in patterns other than exponentially. Hence the numerical solutions are most suitable for these types of problems. In this regard, the finite element method (FEM) is a power tool for solving various multi-disciplinary problems of mechanics including the thermal and mechanical problems in FGMs.

The main issue encountered in using FEM in FGMs is concerned with modelling continuously varying material properties. The simplest way to model a graded material inhomogeneity involves the use of conventional homogeneous elements in successive layers of the mesh, containing own material properties. This leads to a stepwise change in properties along the direction of material gradient. Such models have been already used by a number of researchers, e.g. in [10–12] and they produced reasonable results. However this approach requires a fine mesh to achieve the accuracy, in turn, it leads to excessive computational costs. Moreover, the preparation of input data and adjusting the mesh of different gradation regions is quite cumbersome in this approach as well. Thereby, it is important to include the gradient of material properties into the model at the element level so that the accuracy would be retained when coarser meshes are used. Recently such finite elements (FE), called as graded finite elements have been developed and applied for solving some problems involving FGMs. Authors in [13] proposed a 2-D graded finite element with material properties evaluated directly at the Gauss points. An alternative graded element was developed in [14], a fully isoparametric element formulation that interpolates material properties at each Gauss point from the nodal values, using the same shape functions as the displacements was proposed there. In [15], authors studied thermal stresses due to uniform temperature change in a FGM coating using graded elements, where inhomogeneity was specified at the Gauss points. The transient heat transfer analysis of FGMs using adaptive precise time integration and isometric graded finite elements was carried out in [16].

Although commercially available finite element codes have become popular among engineers and researchers due to their versatility, sufficient accuracy and their ease in use, they pose a challenge to implement variations of material properties in FE models. In [17] authors proposed an original way to assign material gradients within the ANSYS code. They provided the FE model with an initial temperature distribution that matches the desired temperature-dependent material properties. It allowed them to examine a fracture problem in a FGM plate. However this technique is not suitable for thermo-mechanical analyses. In [18] the authors have used the ABAQUS software to calculate the crack-tip field in a cracked FGM plate under mechanical loading. To assign a variation of material properties to the FE model, authors programmed a mechanical law within the user-defined subroutine UMAT. The same approach for developing a graded element has been reported in [19], where the authors applied that element for analysing the pavement behaviour. Moreover, the authors showed that graded element gives a far greater modelling accuracy in comparison with a homogeneous element for a FGM plate under a mechanical load. The same engineering package has also been used for solving heat conduction problems in simple FGM structures in [20]. The authors incorporated gradually varying thermal properties into finite element models of the FGM structures using the user-defined subroutine UMATHT available in ABAQUS for thermal analyses. In [21] the analysis of heat transfer and thermal stresses in metal/ceramic thermal barrier coatings of turbine blades has been carried out using the abilities of the both numerical packages mentioned above, but graded finite elements were not used. The numerical results and analytical solutions on distributions of the temperature and thermally-induced stresses in a FGM plate under thermal shock loading have been obtained in [22]. Those calculations were carried out using the ABAQUS code, where material gradients were programmed by using a combination of mechanical and thermal subroutines of this package. The same user-defined subroutines have been employed by the authors in [23] to carry out the crack propagation analysis with the XFEM approach available in ABAQUS for a FGM strip under thermal shock. More recently in [24] the same authors applied this modelling technique for computing the thermal stress intensity factor in the FGM strip. Also the XFEM has been used for analysing transient thermal shock fracture of functionally graded piezoelectric materials in [25]. The authors in [26] have studied a quasi-static fracture of FGM plates. They modelled the crack growth in plates subjected to mechanical loads by using the XFEM within ABAQUS. In that research, the implementation of material gradients into the models of plates has been achieved by programming the material properties as functions of selected field variables within the USDFLD subroutine instead of using the material user-defined routines. Thermal cracking of a FGM plate undergoing thermal shock has been examined in [27,28]. The authors employed the virtual crack propagation technique available in ABAQUS that had been earlier applied to fracture analysis of laminated composites in [29] to simulate the crack growth in the FGM plate. They incorporated gradients of thermal and mechanical properties of the plate into the model via the combination of subroutines mentioned above. This allowed them to take into account gradual variations of the Poisson's coefficient and the mass density in the calculations that has not been done in the previous works.