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Stochastic multi-scale finite element based reliability analysis for laminated composite structures

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ABSTRACT

This paper proposes a novel multi-scale approach for the reliability analysis of composite structures that accounts for both microscopic and macroscopic uncertainties, such as constituent material properties and ply angle. The stochastic structural responses, which establish the relationship between structural responses and random variables, are achieved using a stochastic multi-scale finite element method, which integrates computational homogenisation with the stochastic finite element method. This is further combined with the first- and second-order reliability methods to create a unique reliability analysis framework. To assess this approach, the deterministic computational homogenisation method is combined with the Monte Carlo method as an alternative reliability method. Numerical examples are used to demonstrate the capability of the proposed method in measuring the safety of composite structures. The paper shows that it provides estimates very close to those from Monte Carlo method, but is significantly more efficient in terms of computational time. It is advocated that this new method can be a fundamental element in the development of stochastic multi-scale design methods for composite structures.

1. Introduction

Typical composite components are laminates comprising layers of fibre reinforced composite laminae, each of which are made of fibres embedded in matrix. The assembly of the fibres and matrix materials to create a lamina, as well as the lay up and curing of laminae, is a complicated process and may involve a lot of uncertainty. Therefore, the material properties of a composite laminate are random in nature. Sources of significant uncertainty include: variations in volume fractions of fibre and matrix, voids in the matrix and between fibres and matrix, imperfect bonding between constituents, cracks, fibre damage, random and/or contiguously packed fibres, misaligned fibres, temperature effects, non-uniform curing of the matrix material, residual stresses, etc. Uncertainties in these factors propagate to a larger scale and are reflected in variability of the stiffness and strength that characterise the overall structural behaviour [1–7]. Consequently, high safety factors of the order

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of 8–10 [8] are introduced in current deterministic based structural design, thereby not taking full advantage of composite materials. These issues may be addressed in a probability based design context [9–11] equivalent to Eurocode, for example.

The reliability index is one of the widely accepted indicators to measure safety of engineering structures subject to uncertainties [12]. The reliability analysis of composites involves several key issues [13–15]. First, many parameters may need to be considered as random variables, but they can be associated with different length scales: micro-scale (constituent level, fibre/matrix), meso-scale (ply level), or macro-scale (component level) [16]. Hence, the choice of uncertainties will directly determine which mechanical model should be used in the structural analysis stage. For instance, a micro-mechanical model is needed when considering micro-scale parameters as random variables. Second, composite materials display a wide variety of failure mechanisms due to their complex structure and manufacturing process, and a range of failure criteria have been proposed, such as maximum stress/strain criteria and polynomial criteria (for instance, Tsai–Wu criteria [17]). Despite extensive research, such as the well known three stages of World Wide Failure Exercises [18–20], a complete and validated methodology for predicting the behaviour of composite structures including the effects of damage has not yet been fully achieved. Qualitative evaluations [13,21,22], quantitative experimental comparisons [23], and numerical comparisons [24] have been made for deterministic failure criteria and quantitative comparison of their performance considering uncertainty was reported in [25], with broad differences. Hence, the choice of failure criterion to establish the limit state function is critical to conduct reliability analysis. Third, for some relatively simple composite structures, analytical formulations have been developed [26–28], while finite element reliability analysis methods are necessary to handle more complex structures [29]. Finally, the essence of reliability analysis is to calculate the failure probability of structural components or systems, which is expressed by the convolution integral. It is impossible to calculate the integration directly due to its multi-dimensionality, and therefore numerical methods have to be used. Monte Carlo simulation (MCS) is a straightforward option [30]. The accuracy of MCS directly depends on the number of simulations, and it is recommended to conduct at least N simulations [31], where $N = -\ln(1-C)/p_f$ with p_f is the expected failure probability and C is a given confidence level (normally 95%), to obtain a sufficiently accurate estimate of p_f. However, the failure probability is generally a small value of the order of 10^{-5}, e.g. Eurocode requires $\beta \geq 3.8$ or $p_f \leq 7 \times 10^{-5}$, and implementing thousands of simulations are thus extremely time consuming, in particular when involving finite element calculations for complex structural systems. Approximation methods such as First-Order Reliability Method (FORM) and Second-Order Reliability Method (SORM) have become popular alternatives due to their efficiency. Recently, new reliability methods have been adopted or proposed to conduct reliability analyses for composite structures, such as Artificial Neuronal Networks [32,33]. Nakayasu et al., [25,34] compared different structural reliability analysis methods for composites and stated a preference of the FORM method.

It has long been recognized that laminate stress analysis and lamina failure criteria are two critical elements in the analysis of composite laminates. In the past years, various homogenization methods have been developed in order to determine the macroscopic material properties of a heterogeneous material from its constituents. However, most of the previous work on reliability analysis for composites had focused only on meso-scale parameters such as ply material properties. It has been widely accepted that the mechanical behaviour of composites are strongly affected by their microscopic variations in material properties [16,35] and homogenization methods have proven to be capable of predicting accurately the effective material properties. Accordingly, the combination of probabilistic modelling and micromechanics seems to be an appropriate approach to achieve the consistent characterisation of composites behaviour [36,37]. The accuracy of structural reliability estimation may be improved by using multi-scale methods [38,39]. Although stochastic multi-scale finite element methods have been developed for many years by various researchers, e.g. [40–42], except in a few cases [43–48], multi-scale modelling of such materials has been limited to purely deterministic analyses.

The objective of the present study is to propose and evaluate a multi-scale finite element based reliability analysis approach in order to address some of the above mentioned challenges. This approach combines a state-of-the-art computational multi-scale homogenization method with composite mechanics and structural reliability analysis. It enables uncertainties in both microscopic and macroscopic parameters to be considered. Stochastic structural responses are obtained from a stochastic multi-scale finite element method, which establishes the relationship between structural responses and microscopic random variables. The commonly used FORM and SORM are coupled with the multi-scale finite element method to conduct reliability analyses. Numerical studies are performed to illustrate the procedure for the reliability analysis of composite structures and to demonstrate the efficiency and accuracy of the proposed approach. The proposed method will serve as a fundamental component in the development of stochastic multi-scale design method for composite structures. Innovative construction and building technologies are usually driven by the developments of new constructional materials and/or structural forms [49,50].

2. Stochastic multi-scale finite element method

2.1. Stochastic homogenization method for composite materials

First we will summarize the basic assumptions and the final formulae of the probabilistic homogenization method for the estimation of effective elastic moduli developed by the authors in [42]. The class of homogenization-based multi-scale constitutive models employed in the present study is characterised by the assumption that the strain ($\bar{\boldsymbol{\varepsilon}}$) and stress tensor ($\bar{\boldsymbol{\sigma}}$) at a point of the so-called macro-continuum are the volume average of their respective microscopic counterpart fields