Applied Mathematical Modelling 000 (2016) 1–27

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Undamped eigenperiods of a sea-based gravity monotower

O.M. Faltinsen[a], A.N. Timokha[a,b,*]

[a] Centre for Autonomous Marine Operations and Structures, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
[b] Institute of Mathematics, National Academy of Sciences of Ukraine, Tereschenkivska 3, Str., 01601 Kiev, Ukraine

ARTICLE INFO

Article history:
Received 22 August 2015
Revised 26 February 2016
Accepted 13 April 2016
Available online xxx

Keywords:
Monotower
Coupled dynamics
Sloshing
Variational methods

ABSTRACT

An analytically approximate method was proposed in [1] to estimate undamped eigenperiods of a sea-based gravity monotower by accounting for sloshing inside the shaft. These results are generalised to include the soil feedback and the inertia moment of the top rigid body as well as to describe the external hydrodynamic loads using the linear three-dimensional potential flow theory of an incompressible liquid. A mathematical model of the multicomponent mechanical system is presented. The virtual work principle is used to express the Euler–Bernoulli governing equation and all fluid-structure dynamic transmission conditions. Numerical examples of the eigenperiods *versus* geometric and physical input parameters typical for the Draugen platform and some monopiles are given. The highest eigenperiod of the horizontal vibrations belongs to experimentally-known ranges. These eigenperiods increase with increasing mass, radius of gyration and mass centre of the top body; they also increase with the shear modulus of the soil. Three classes of eigenmodes are detected. They express dominant character of structural vibrations and sloshing, respectively, or a mixed type. Sloshing is less important for existing monopiles.

1. Introduction

Sea-based gravity monotowers are used in the oil/gas production and the wind energy industry. The tower shaft is normally filled with water [1–6]. A rigid body (operational platform, rotor, nacelle, etc.) is installed at the tower top. Horizontal and vertical structural vibrations are affected by the soil as well as the exterior and interior hydrodynamic loads. Excitations could be related to incident (periodic or/and impact) wave loads and Earthquake. The two main differences of our monotower problem relative to the elevated water tank problem [7,8] are that we have to consider hydrodynamic loads associated with the outer water and that sloshing loads are not applied to the tower top but the shaft wall. The latter is like in the fuel rocket problem [9,10].

Pursuing a *non-phenomenological* theory of the eigenvalue problem of a sea-based gravity monotower, [1, Section 5.4.5], proposed an approximate mechanical/mathematical model in which: (i) soil beneath the monotower is rigid, (ii) a mass point (*not rigid body!*) is attached to the tower top, (iii) strip theory with frequency-independent added mass described the external hydrodynamic loads in the undamped eigenvalue problem. The present paper derives a more sophisticated mechanical/mathematical model assuming that (i*) the elastic soil feedback matters, (ii*) a rigid body (*not mass point!*) is

* Corresponding author at: Centre for Autonomous Marine Operations and Structures, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway. Tel.: +47 73595524.
E-mail addresses: odd.faltinsen@ntnu.no (O.M. Faltinsen), alexander.timokha@ntnu.no (A.N. Timokha).

http://dx.doi.org/10.1016/j.apm.2016.04.003

attached to the tower top, (iii*) the external hydrodynamic loads follow from considering the linear three-dimensional water wave problem with body interactions. *Excitations are not considered*. Emphasis is on a theoretical estimate of *undamped eigenperiods*. Numerical examples deal with the Draugen monotower and a sample gravity monopiles from the wind energy industry.

Elastic, rigid, and hydrodynamic elements of a multicomponent mechanical system modelling a sea-gravity monotower are introduced in Section 2. Its central element is the so-called inverted elastic pendulum [7,8], i.e. a vertical Euler–Bernoulli beam, which is set upon a stiff/hard soil and has a rigid body attached to the top. The axial elastic beam oscillations are neglected since they are characterised by much lower eigenperiods than periods of external wave excitations. The soil elasticity introduces restoring forces and moments applied to the lower beam end. The beam is characterised by noncontinuous physical and geometrical input parameters and interacts with the inner (in shaft) and external (sea water) liquids. Soil, exterior and interior water are coupled with the beam motions but not directly with each other. As a consequence, the soil-related response and the hydrodynamic loads become only functions of the structural deflection and velocity. The virtual work principle plays the role of a variational governing equation of the beam. By assuming the coordinate axes are parallel to the principal axes of inertia of the top rigid body decouples the unknown variables so that one can analyse, independently, the *free unforced horizontal structural vibrations* in the two vertical coordinate planes as well as the vertical unforced structural motions (as a rigid body).

Section 3 evaluates the undamped eigenperiod of the *vertical* vibrations. For an *almost cylindrical* tower that is typical for the sea-based gravity monopiles of the wind turbine industry, the external hydrodynamic loads due to the vertical tower vibrations can be neglected. However, they must be included for the Draugen platform, which has a massive deeply submerged caisson. Computations show that the corresponding nondimensional added mass is weakly frequency-dependent. For the Draugen platform, including the added mass effect increases the undamped eigenperiod for vertical motions by about 30% (e.g., from 0.375 to 0.43 s).

In Section 4, the aforementioned decoupling of the unknown variables makes it possible to concentrate on undamped horizontal eigenoscillations in a vertical coordinate plane. The virtual work principle from Section 2 derives a variational governing equation of the corresponding eigenvalue problem restricted by three functional constraints, which express the hydrodynamic loads associated with the external flow, the Stokes–Joukowski potential (assuming frozen inner free surface) and sloshing. The constraints appear as three linear operators defined on the trial structural eigenmodes. When the trial eigenfrequencies belong to a subset of the natural sloshing frequencies, the sloshing-related functional constraint becomes mathematically undefined. We select three classes of eigenperiods and modes. The first class ($\{T_k\}$ and $W^{(k)}(z)$) neglects sloshing, i.e. what we call the *structural* eigenperiods and modes. Describing them implies finding the structural eigenoscillations affected by the external frequency-dependent added mass and the frequency-independent internal added mass associated with the Stokes–Joukowski potential (see, more details on the Stokes–Joukowski potential in [1, chapter 5]). The second class of eigenperiods $\{T_{sk}\}$ is the *natural sloshing periods*. The third class corresponds to the *coupled structure-and-sloshing eigenoscillations* with eigenperiods $\{T_{ck}\}$, structural $W_c^{(k)}$ and sloshing-related $\phi_{W_c^{(k)}}$ eigenmode-components. Relationships between these three classes versus geometric and physical parameters are described by using a projective scheme based on the Trefftz method for the hydrodynamic problems and a special functional basis for the structural eigenmodes $W^{(k)}(z)$ (or $W_c^{(k)}$). The basis provides a natural classification of the structural motions distinguishing, in particular, the rigid-body beam motions and the beam deflections considered in [1, Section 5.4.5]. The convergence is reported in Section 4 (regarding the Trefftz method) and in Section 5 (for the entire projective scheme).

Section 5 collects and discusses a series of numerical examples associated with the Draugen platform and a sample gravity monopiles whose geometric and physical parameters are provided in Appendices C and B, respectively. The section starts with analysing assumption (iii). We show that there are clear three-dimensional flow effects at the free surface and at the caisson of the Draugen platform. The frequency-dependency matters only near the mean sea surface.

In Section 5.2, numerical studies of the highest eigenperiod for the Draugen monotower are conducted. A good agreement with the full-scale observed period range is documented. Sloshing cannot be neglected. Furthermore, even though the model assumes the caisson as a piece of the Euler–Bernoulli beam, computations confirm as expected that the caisson moves like a rigid body and the speculatively-taken Young's modulus for this beam piece weakly affects that.

Evaluating a sample gravity monopiles proposed in Appendix B to fit a future, much more massive wind-energy construction, one can expect $T_1 \approx T_{s1}$ for certain soil characteristics and parameters of the top-attached rigid body (here, rotor and nacelle). Numerical studies in Section 5.3 establish qualitatively the same behaviour of the monopiles as it was described in Section 5.2 (the Draugen monotower). However, computations show that sloshing loads can, generally, be neglected. Another matter would be if a Tuned Liquid Damper is installed close to the top of the monopiles.

2. Mechanical/mathematical model

2.1. Introductory remarks

A mathematical model of the considered multicomponent engineering system is presented in a dimensional form. The model can be rewritten in a nondimensional form after choosing the characteristic size and time as the tower radius at the mean free surface and the highest eigenperiod of the corresponding cantilever beam.