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A new generalized burn-in procedure for items in stochastically evolving population

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ABSTRACT

In practice, the reliability performance of items in operation strongly depends on the operational history or the working environment. Thus, most often, the population stochastically evolves depending on it rather than being static. For example, the failure and repair history strongly affects the future reliability performance of the item in operation. In this case, even though the population of items initially starts from a homogeneous one, it gradually evolves to a heterogeneous one. Until now, most studies on burn-in have been performed assuming that the population is static. In this paper, a new generalized burn-in procedure for items in stochastically evolving population is developed and its stochastic properties are analyzed. Based on the obtained theoretical results, the optimal burn-in procedure which balances two criteria is studied.

1. Introduction

Recently, items are becoming more and more repairable due to the complexity of the functions required for them. In most of the studies on burn-in procedure for repairable items, minimal repair has been assumed in the literature [1–6]. The corresponding population of items in this case is *'static'* in the sense that the future reliability performance of the items in operation does not depend on the operational history (i.e., the failure and repair history), due to the independent increments property of the nonhomogeneous Poisson process (NHPP). In this case, the traditional 'time-based burn-in' has been applied, i.e., *those items which fail during the burn-in procedure are minimally repaired and, after the burn-in time b, the items are put into the field operations* [1,2,4,5,7]. When the items have a bathtub-shaped failure rate, which exhibits initially decreasing failure rate, the general goal of the above time-based burn-in is to shift the failure rate function to the left and to avoid in this way its initially large failure rate [1,2,4,5,7]. However, in practice, the future reliability performance of the item in operation strongly depends on the operational history or its working environment. Thus, most often, the population *stochastically evolves* depending on it rather than being static. For example, the detailed failure and repair history of the item strongly affects the future reliability performance. In this case, even though the population of items initially starts from a homogeneous one, it gradually evolves to a heterogeneous one.

In this paper, we develop a new generalized burn-in procedure for items in stochastically evolving population. By the new burn-in procedure proposed in this paper, the items after burn-in are eliminated based on additional information on the evolving operational history during the burn-in interval. The effect of burn-in on the population reliability characteristic is stochastically analyzed and optimal burn-in is discussed based on the obtained theoretical results. The paper is organized

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as follows. In Section 2, a stochastic model for modelling evolving population of items is introduced and the generalized burn-in procedure is proposed. The effect of the burn-in procedure on the population reliability characteristic after the burn-in is stochastically analyzed. Furthermore, the corresponding stochastic failure processes after the burn-in are characterized. In Section 3, optimal burn-in which takes into account both the quality and cost aspects is studied. Finally, in Section 4, concluding remarks are given.

2. A new generalized burn-in procedure for evolving population

In this paper, we consider repairable items. Suppose that the future reliability performance of the item in operation depends fully or partially on the operational history $\mathbf{H}_t$, which is observable, and each item conveys it. The items are operated during the burn-in time b. Those items which fail during the burn-in procedure are repaired and, after the burn-in time b, the item is put into the field operation. Under this situation, as the future reliability performance of items may dramatically change depending on its operational history, it would be reasonable to eliminate items having $\mathbf{H}_b$ which indicates poor reliability performance in the field. Thus, let us consider the following generalized burn-in procedure:

- *Generalized burn-in procedure based on* $\mathbf{H}_b$

The items are operated during the burn-in time b. Those items which fail during the burn-in procedure are repaired. After the burn-in time b, if $\mathbf{H}_b \in \mathbf{S}_b$ then the item is eliminated; otherwise, it is put into the field operation.

Thus, $\mathbf{S}_b$ corresponds to the set of histories which corresponds to the elimination of the corresponding burned-in item. Under the above generalized burn-in model, not only the burn-in time b, but also the set $\mathbf{S}_b$ should be determined. As the operational history, one may consider any kinds of information process on the system performance such as the deterioration history, the degradation process periodically observed, the failure and repair history, etc. In this paper, we will consider a stochastic model for modelling evolving population of items, where the items' future reliability performances depend on their failure and repair histories.

2.1. A stochastic model for an evolving population

Recently, in Ref. [8], a new counting process, called the 'Generalized Polya Process' (GPP), was defined based on the concept of stochastic intensity and, based in it, a new repair process was defined. Now we briefly introduce the GPP. Let $\{N(t),\ t \ge 0\}$ be an *orderly* point process and $\mathbf{N}_{t-} \equiv \{N(u),\ 0 \le u < t\}$ be the history (internal filtration) of the process in $[0, t)$, i.e., the set of all point events in $[0, t)$. Observe that $\mathbf{N}_{t-}$ can equivalently be defined in terms of $N(t-)$ and the sequential arrival points of the events $T_0 \equiv 0 \le T_1 \le T_2 \le \cdots \le T_{N(t-)} < t$ in $[0, t)$, where T_i is the time from 0 until the arrival of the ith event in $[0, t)$. The point processes can be mathematically described by using the concept of the stochastic intensity (the intensity process) λ_t, $t \ge 0$ [9,10], which is defined as [8,11,12]:

$$\lambda_t = \lim_{\Delta t \to 0} \frac{\Pr[N(t, t+\Delta t) = 1 | \mathbf{N}_{t-}]}{\Delta t} = \lim_{\Delta t \to 0} \frac{E[N(t, t+\Delta t) | \mathbf{N}_{t-}]}{\Delta t},$$

where $N(t_1, t_2)$, $t_1 < t_2$, represents the number of events in $[t_1, t_2)$. Examples of stochastic intensities for several popular counting processes can be found in Ref. [8]. The definition of the GPP is as follows.

Definition 1. Generalized Polya Process (GPP)

A counting process $\{N(t),\ t \ge 0\}$ is called the Generalized Polya Process (GPP) with the set of parameters $(\lambda(t), \alpha, \beta)$, $\alpha \ge 0$, $\beta > 0$, if:

(i) $N(0) = 0$;
(ii) $\lambda_t = (\alpha N(t-) + \beta)\lambda(t)$.

As mentioned in Ref. [8], the GPP with $(\lambda(t), \alpha = 0, \beta = 1)$ reduces to the NHPP with the intensity function $\lambda(t)$ and, accordingly, the GPP can be understood as a generalized version of the NHPP.

Based on the GPP, we will now define a new type of repair, which is called the 'GPP repair'. In the following discussions, we assume that the time for a repair is negligible. Define $\{N(t),\ t \ge 0\}$ as the failure process of the item with its failure rate $\lambda(t)$ which undergoes a type of repair on each failure, where $N(t)$ is the total number of failures in $(0, t]$.

Definition 2. GPP repair

For an item with its failure rate $\lambda(t)$, a repair type is called the 'GPP repair' with the parameter α if $\{N(t),\ t \ge 0\}$ is the GPP with the parameter set $(\lambda(t), \alpha, 1)$.

Thus, under the GPP repair process, the corresponding stochastic intensity is specified as:

$$\lambda_t = (\alpha N(t-) + 1)\lambda(t). \quad (1)$$

Under the stochastic model in (1):

"*The population of items undergoing the GPP repair process initially starts from a homogeneous one, but it evolves to a heterogeneous one*".