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The SEIQS stochastic epidemic model with external source of infection

Julia Amador*

Faculty of Statistical Studies, Complutense University of Madrid, 28040 Madrid, Spain

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ABSTRACT

This paper deals with a stochastic epidemic model for computer viruses with latent and quarantine periods, and two sources of infection: internal and external. All sojourn times are considered random variables which are assumed to be independent and exponentially distributed. For this model extinction and hazard times are analyzed, giving results for their Laplace transforms and moments. The transient behavior is considered by studying the number of times that computers are susceptible, exposed, infectious and quarantined during a period of time $(0, t]$ and results for their joint and marginal distributions, moments and cross moments are presented. In order to give light this analysis, some numerical examples are showed.

1. Introduction

The first work about epidemiology application to computer viruses is due to Murray in 1988 [1], although he does not describe any model. Kephart and White [2,3] have been the pioneers of modeling the spread behavior of computer viruses throughout a Susceptible-Infective-Susceptible (SIS) model. Since then epidemic models have been widely used in order to model the spread behavior of computer viruses by introducing modifications to the simplest models, SIS and SIR (Susceptible-Infective-Recovered). A lot of models applied to computer environment are deterministic, based on ordinary differential equations (ODEs) (see [4–10], for example). Piqueira et al. [6] (see also [7]) deal with a modification of the traditional SIR model with an antidotal population compartment (A). Mishra and Saini [4] take into account a latent period where computers remain in the exposed class (E) before becoming infective (SEIRS epidemic model). Yao et al. [10] implement the quarantine class (Q) to the model (SIQ model), and Mishra and Jha [5] consider a model with exposed and quarantine classes (SEIQRS model); Wang et al. [8] also consider exposed and quarantine compartments and they analyze a more sophisticated SEIQRS model that presents more transitions and rates than the aforementioned SEIQRS model. Recently Yang and Yang [9] have described a new epidemic model by distinguishing between internal or external computers depending on whether they are connected to the Internet or not, and they also consider latent periods for viruses.

Stochastic epidemic models take into account the random nature of population events and they are more appropriate than deterministic models for small populations (see [11–14]). We find different types of stochastic epidemic models applied to computer environment: stochastic differential equation models (see [15]), continuous-time Markov chain models (see [16–20]) and we can also find works focus on inference from a Bayesian perspective (see [21])

* Tel.: +34913943994.
E-mail address: jamador@estad.ucm.es

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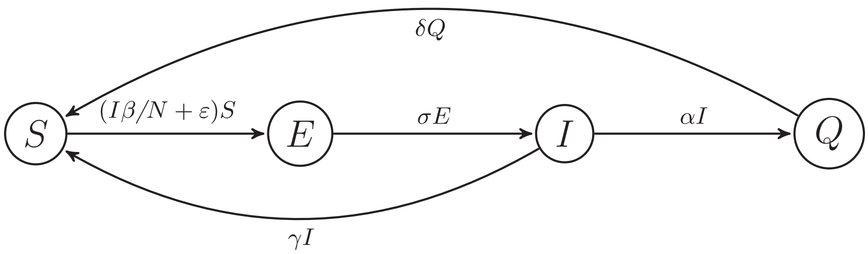

Fig. 1. Compartmental flow chart.

Zhang et al. [15] introduce a random noise in ODEs of a deterministic SEIR model and transform it into a corresponding stochastic differential equation model.

We are concerned with stochastic models that employ continuous time Markov chains (CTMC) for modeling the propagation of viruses. In this setting, we point out that Weiss and Dishon [22] consider a continuous birth-and-death process to describe the SIS model. Wierman and Marchete [20] extend the stochastic SIS model by taking account of reintroduction of computer virus; Okamura et al. [19] propose the stochastic KS model, i.e. the stochastic SIS model with kill signals; the idea of incorporating the kill signals to a stochastic epidemic model is also found in Amador and Artalejo [17]. Amador [18] describes and analyzes the stochastic SIRA model, i.e. the extension of traditional stochastic SIR model by including antidotal computers.

In this paper the interest is focused on the stochastic SIS model which incorporates latent and quarantine periods, i.e. stochastic SEIQS model, and it considers two different sources of infection, by direct contact with an infectious computer or by an external source. The description of this model is given in Section 2 by using a continuous-time Markov chain. Exponential distributions and independence of the involved random periods are two fundamental assumptions of a continuous-time Markov chain that make the probabilistic model tractable, so they are commonly assumed [17–20,22,23]. Moreover, there are some studies with real epidemic data which verify the validity of these assumptions [24,25]. An alternative approach is the block-structured state-dependent event (BSDE), introduced by Artalejo and Gomez-Corral [26], which is helpful to deal with non-exponential correlated flows. This approach has also been used in [16,27]. The problem is that the BSDE version of an epidemic model augments the dimensionality of the original model and hence it can be intractable. For this model, it is interesting to study characteristics related with the first time at which all computers are infected or the first time at which no-one is infected: the hazard time and the extinction time, respectively. These first passage times are analyzed in Section 3. It is also important to know the situation of computers during a fixed period of time and this is done in Section 4. Some numerical examples are presented in Section 5 in order to illustrate the results of previous sections. Finally, Section 6 contains some conclusions of this work.

2. Model description

The stochastic *SEIQS* model is an extension of the classic stochastic *SIS* model for which latent and quarantine periods are considered. More concretely, we deal with a closed population of size N (e.g. N computers) which is partitioned into subclasses of computers, namely susceptible, exposed (infected but not yet infectious), infectious and quarantined (infectious computers which are isolated). In this model we assume two sources of infection, internal infections caused by transmission from any infectious computer in the population and external infections coming from outside the computer network. When a susceptible computer is infected, there is a period of time during which this computer does not transmit the infection (latent period), after this time it becomes infectious and it can be isolated in order to avoid contagion. After a time, an infectious computer is recovered but it does not acquire immunity and it becomes susceptible immediately. Let $S(t)$, $E(t)$, $I(t)$ and $Q(t)$ be, respectively, the number of susceptible, exposed, infectious and quarantined computers at time t, where one of them can be expressed in terms of the other three, e.g. $Q(t) = N - S(t) - E(t) - I(t)$.

Let us assume an initial condition $(S(0), E(0), I(0)) = (i_0, j_0, k_0)$, with $k_0 \geq 1$ and $i_0 + j_0 + k_0 \leq N$; i.e. at time $t = 0$ there are at least one infected computer. If the state is (i, j, k) at time t, the possible transitions are as follows: towards the state $(i-1, j+1, k)$ when a susceptible computer is infected, with rate $(k\beta/N + \varepsilon)i$ ($i > 0$), being ε the individual external infection contact rate and β the internal infection contact rate; other possibility is to go to the state $(i, j-1, k+1)$ which occurs when an exposed computer becomes infectious, with rate σj ($j > 0$), σ is called individual incubation rate; the transition to the state $(i, j, k-1)$ happens when an infectious computer is isolated, with rate αk ($k > 0$), α is called individual quarantine rate; other transition is to the state $(i+1, j, k-1)$ when an infected computer is recovered and then susceptible, with rate γk ($k > 0$), γ is called individual recovery rate from infection; and the last one is to the state $(i+1, j, k)$ when a quarantined computer is recovered becoming susceptible, with rate $\delta(N-i-j-k)$ ($(N-i-j-k) > 0$), being δ the individual recovery rate from quarantine. Fig. 1 illustrates these transitions.

By assuming exponential distributions and independence of the involved random periods, the process $\mathbf{X} = \{(S(t), E(t), I(t)); t \geq 0\}$ is a tridimensional *CTMC* on the state space $\mathbf{S} = \{(i, j, k) : 0 \leq i \leq N,\ 0 \leq j \leq N,\ 0 \leq k \leq N,\ i + j + k \leq N\}$. The infinitesimal generator of this *CTMC*, $\mathbf{Q} = (q_{(i,j,k),(i',j',k')} : \{(i, j, k), (i', j', k')\} \in \mathbf{S})$, is a square matrix of

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