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In-plane stress analysis of dissimilar materials with multiple interface cracks

M.M. Monfared [a], M. Ayatollahi [b,*], R. Bagheri [c]

[a] Department of Mechanical Engineering, Hashtgerd Branch, Islamic Azad University, Alborz, Iran
[b] Faculty of Engineering, University of Zanjan, P.O. Box 45195-313, Zanjan, Iran
[c] Department of Mechanical Engineering, Karaj Branch, Islamic Azad University, Alborz, Iran

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ABSTRACT

In the present paper, the behavior of an interface crack in two bonded dissimilar materials subjected to in-plane loading is studied. First, dislocation solution is presented to construct integral equations for dissimilar medium weakened by several interface cracks. These equations are of Cauchy singular type at the location of dislocation, which are solved numerically to obtain the dislocation density on the faces of the cracks. The dislocation solution is then applied to calculate the mixed mode stress intensity factors of multiple interface cracks. The effects of nonhomogeneity parameter on the stress intensity factors are investigated.

1. Introduction

Bimaterials are extensively used in many engineering material system, such as composite structures, electronic packaging, and thin film constructions. The problem of determining the stress intensity factors of interface cracks is of considerable importance in the design of safe structures. It has been shown that for interface cracks the material properties play an important role in their fracture mechanics behavior. From microelectronics to structural engineering many physical applications require joining of two dissimilar materials. Generally the main function of the "interface" thus formed is the effective transfer of some flux quantity such as stress, heat or electricity. In 1960 s, elastic solutions around an interface crack tip were derived and the definition of the stress intensity factors was discussed by Williams [1], Erdogan [2,3], England [4] and Rice and Sih [5]. In these solutions, the oscillation of stresses and the overlap of crack surfaces appear in the vicinity of a crack tip. The interface crack behavior in dissimilar anisotropic composites under mixed mode condition was studied by Wang and Choi [6]. Delale and Erdogan [7] investigated interface crack problem between two homogeneous and nonhomogeneous half-plane. In this study, they obtained the integral equations and the asymptotic behavior of the stress state near the crack tip. Two bonded dissimilar homogeneous elastic half-planes under mixed mode loading were considered by Delale and Erdogan [8]. They obtained modes I and II stress intensity factors, the energy release rate and the direction of a probable crack growth. Shih and Asaro [9] investigated the near tip elastic-plastic fields of a crack on a bimaterial interface. They found that the oscillations of stresses and the overlap of crack surfaces predicted by the original solutions vanish in the elastic-plastic field in the vicinity of the crack tip in the case where tensile load is dominant. Three-dimensional finite element computations were used to investigate the stress field near the interface crack by Nakamura [10]. Hutchinson and Suo

* Corresponding author. Tel.: +98 24 3305 2488; fax: +98 24 3305 3204.
E-mail address: mo.ayatollahy@gmail.com (M. Ayatollahi).

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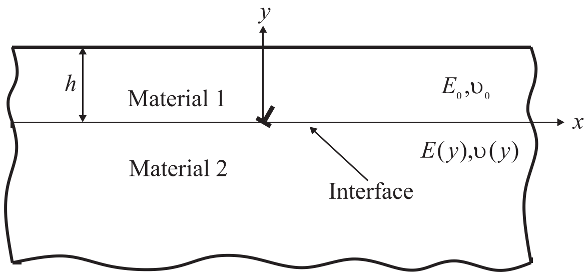

Fig. 1. Schematic of dislocation between two dissimilar materials.

[11] reviewed the stress intensity factor of the mixed mode fracture of an interface crack. Chen and Erdogan [12] investigated the problem of the interface crack in a nonhomogeneous coating bonded to a homogeneous substrate. The problem of a penny-shaped crack in homogeneous dissimilar materials bonded through an interfacial region with graded mechanical properties was studied by Ozturk and Erdogan [13]. The symmetric mode I crack problem in an inhomogeneous orthotropic medium was examined by Ozturk and Erdogan [14]. It has been shown that in the mode I crack problem, for an orthotropic inhomogeneous medium, the Poisson's ratio has only a negligible influence on the stress intensity factors but the effect of the material inhomogeneity was quite significant. Ikeda et al. [15] presented the concept of the stress intensity factors of an interface crack between dissimilar materials, and various types of specimens are tested experimentally for investigating the mixed mode facture toughness criterion of an interface crack. The mixed mode crack problem for an inhomogeneous orthotropic medium was treated by Ozturk and Erdogan [16]. It was found that generally the stress intensity factors increase with increasing materials inhomogeneous parameter and with decreasing stiffness ratio. Huang and Kardomateas [17] obtained the modes I and II stress intensity factors in a bimaterials half-planes. Wang and Zhong [18] analyzed the problem of a crack sliding interface between anisotropic bimaterials. They presented closed form solutions of the stress and displacement in the entire domain. Zhou et al. [19] solved the problem of a Griffith crack at the interface of a layer bonded to a half plane. Effects of the materials anisotropy on the mixed mode stress intensity factors of a central crack were studied by Long and Delale [20]. It has been shown that crack length, orientation and the nonhomogeneity parameter of the layer have significant effect on the fracture of the FGM layer. Ma et al. [21] analyzed the mixed mode crack problem for a functionally graded orthotropic medium under time-harmonic loading. Mixed-mode fracture analysis of an orthotropic functionally graded material (FGM) coating was considered by Dag and Ilhan [22]. Pant et al. [23] analyzed the problem of interface cracks located between two dissimilar materials. They obtained the mixed mode stress intensity factors for bimaterial interface cracks. The problem of an interface crack for a homogeneous orthotropic strip sandwiched between two different functionally graded orthotropic materials subjected to thermal and mechanical loading, was examined by Ding et al. [24]. Recently, the stress intensity factor in orthotropic bimaterial interface cracks under mixed mode loads was obtained by Zhao et al. [25]. Fan et al. [26] considered the mixed-mode Dugdale model to solving the plastic zone size and crack tip opening displacement for arc-shaped interface crack. In another paper Fan et al. [27] solved the problem of an arbitrarily oriented Zener–Stroh crack near a bimaterial interface.

Monfared et al. [28], solved the problem of functionally graded orthotropic half-plane with multiple cracks. The stress intensity factors were obtained for the cracks with different configurations and arrangements. In another paper, Monfared and Ayatollahi [29] obtained the mixed mode stress intensity factors for multiple cracks in nonhomogeneous orthotropic planes. Problems addressed in the above last two papers involved cracks whose tips were wholly surrounded by a material.

In this study, we examine cracks whose tips lie on the interface between dissimilar media, in this case we encounter familiar Cauchy type singularities for the stress components at the dislocation location. The primary objective of this study is to apply the distributed dislocation technique for the stress analysis of multiple cracks lying along the interface between two dissimilar materials under in-plane traction. The dislocation solutions are then used to obtain singular integral equations for the dislocation density on the surface of multiple interface cracks. The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the cracks surfaces. Finally, numerical calculations have been carried out to show the influences of material properties and crack size upon the modes I and II stress intensity factors.

2. Formulation of the problem

The geometry of the problem is shown in Fig. 1. It is assumed that homogeneous strip (material 1 occupying $0 < y < h$) is bounded to a non-homogeneous half plane (material 2 occupying $y < 0$). In the graded half-plane, material properties vary in y-direction. In this section, the stress analyses of mediums containing Volterra-type climb and glide edge dislocations is taken up.

Under the condition of in-plane deformation, Hooke's law for two mediums have following strain–stress relationships:

$$\varepsilon_{xx}(x,y) = \frac{1}{E_0}[\sigma_{xx}(x,y) - \upsilon_0 \sigma_{yy}(x,y)],$$