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Rayleigh wave dispersion in an irregular sandy Earth's crust over orthotropic mantle

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ABSTRACT

The present paper presents a study of Rayleigh wave propagation in a sandy crustal layer of the Earth lying over an orthotropic mantle with irregular boundary surfaces under the influence of initial stress, gravity field and rigid boundary plane. An attempt has been made to study the dynamics of the individual medium and derive the displacement of the wave for both the media separately. Suitable boundary conditions have been imposed to derive the dispersion equation in closed form. In general, the obtained dispersion equation is complex, which on separating into real and imaginary parts, leads to two different dispersion equations, of which we are neglecting the imaginary part being damping in nature. The real part contains complete information regarding corrugation parameter, wave number, phase velocity, sandy parameter, initial stress and gravity field. The results for irregular boundaries have been obtained in three cases and presented using graphs. It has been found that the initial stress and sandy parameter have a significant effect and remarkable bearing on the phase velocity of Rayleigh surface wave. Corrugation parameters also have a dominant impact on the propagation of Rayleigh wave. This study may be of great help in the field of industrial engineering, submarine structures, civil engineering, aerospace, chemical pipe and metallurgy as well as geophysics.

1. Introduction

The study of surface waves and their characteristics are often carried out to identify the properties and structure of the Earth's interior. Waves that propagate along the surfaces or interfaces are very important to seismologists and earthquake engineers in understanding the damage caused due to earthquakes. Keeping in view the fact that the seismic waves that are generated during earthquake encounter mountain basins, mountain roots, salt and ore bodies in their paths, such irregularities certainly affect the surface wave propagation. In fact, the boundaries of the Earth's crusts and their various layers can never be flat but are always irregular up to some extent. The corrugated surfaces or the irregular undulation of the boundary surface contribute to the energy distribution of reflected and refracted waves. These imperfect interface or corrugated surfaces, in turn, may be considered to be made up of trigonometric functions. As the analytical treatment of the irregularities of the surface entails formidable mathematical difficulties, researchers particularly in this area, concentrated their effort with considerable success in considering the cases of slightly curved surfaces of different types. Due

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to the vital role of irregularities and initial stresses on the surface waves such as Rayleigh wave, several papers concerning Rayleigh wave propagation in various elastic medium have been published. Stonely [1] investigated the transmission of Rayleigh waves in a heterogeneous medium while Newlands [2] studied in details about the Rayleigh waves in a two layer heterogeneous medium. Rayleigh wave with nonlinear damping has been studied by Fan [3], whereas Sharma and Sharma [4] came out with an interesting paper on modeling of thermoelastic Rayleigh waves in a solid underlying a fluid layer with varying temperature. Abd-Alla et al. [5] investigated the influence of gravity field and initial stress on the Rayleigh waves propagating in a granular medium. Later, Abd-Alla et al. [6] also presented propagation of Rayleigh waves in generalized magneto-thermoelastic orthotropic material under initial stress and gravity field. Naturally, these concepts lead us to investigate Rayleigh wave in an irregular sandy Earth's crust over orthotropic mantle under the effect of initial stress and gravity field. Effect of magneto-thermo materials, effect of rotation, initial stress and gravity field have also been studied by Abd-Alla [7] while reflection of a plane magneto-thermoelastic wave at the boundary of a solid half space in presence of initial stress has been investigated by Singh and Chakraborty [8]. Dynamic crack growth under Rayleigh wave and formulas for the Rayleigh wave speed was investigated by Slepyan [9] and Vinh and Odgen [10], respectively. Earth being a gravitating medium along with the presence of overburden layers, the presence of gravity field and internal friction of dry sandy material affects the propagation of the Rayleigh waves to a great extent. The acceleration due to gravity g also plays a role in studying the propagation characteristic of Rayleigh surface wave. This motivated us to choose a sandy Earth's crust lying over orthotropic half space under the influence of initial stress. These initial stresses in the mantle may be due to variation of temperature, gravitating pull, slow process of creep and pressure of burden layers.

In fact, study of surface waves such as Rayleigh wave in homogenous, heterogeneous and layered media of various natures has been of central interest to theoretical seismologists until recently. Rayleigh waves are dispersive over layered geologies, that is, the phase velocity of a Rayleigh wave depends on its frequency (the relation is called a dispersion curve). Longer wavelength Rayleigh waves penetrate deeper than shorter wavelengths for a given mode and generally have greater phase velocities. The phase velocities of surface waves have to be calculated for many applications, for example in modeling regional Lg and Rg waves (Oliver and Ewing [11]), in inferring the Earth's structures as well as in synthesizing the complete seismogram. References can be made to Pei et al. [12] for his improved work on computation of phase velocities of Rayleigh waves based on the generalized R/T coefficient method. Reference can also be made to Vinh et al. [13], Vishwakarma and Gupta [14], Kazumi et al. [15] and Sharma [16] for their recent outstanding work in the field of Rayleigh wave propagation in various geo-media.

The propagation of waves in orthotropic medium has many applications in various fields of science and technology, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessels, aerospace, chemical pipes and metallurgy. Orthotropic material is one that has different material properties or strengths in different orthogonal directions. In recent times Pal et al. [17] has investigated propagation of Rayleigh waves in an anisotropic layer overlying a semi-infinite sandy medium, where the initial stress and gravity have been ignored. The anisotropic layer was kept flat (without any irregularity) with free boundary plane at the top. Therefore, keeping in view the possibility and applications, the present problem has been formulated to study Rayleigh wave dispersion in a sandy Earth's crust with rigid boundary plane over a compressible orthotropic mantle under the influence of gravity field and initial stress with corrugated boundary surfaces. The upper boundary of the sandy layer and the interface of the two medium have been considered with periodic irregularities and are discussed in detail for all possible cases. Graphs have been plotted for dimensionless wave number against dimensionless phase velocity for various values of initial stress, sandy parameter and corrugation parameter. It has been found that as the initial stress increases, the phase velocity of Rayleigh wave diminishes remarkably showing a clear departure from its static position, while increasing sandiness in mantle along downward direction allows propagation of Rayleigh wave more easily in the upper surface of the Earth. It is also observed that corrugated boundary surfaces also affect the propagation to a greater extent.

2. Geometry of the problem

In the present problem, we have considered a sandy Earth's crust (Medium 1), $M_1 : I_1(x), I_1(x) - H \leq z$ over an initially stressed orthotropic mantle (Medium 2), $M_2 : I_2, I_2(x) \leq z \leq \infty$ as shown in Fig. 1, where H can be assumed as the average thickness of the upper layer, $I_1(x)$ and $I_2(x)$ are continuous functions of x independent of y known as irregular boundaries of the layer and the interface respectively. The sandy layer has been loaded with a rigid boundary at the top thereby not allowing particle to displace from its position. The x-axis and y-axis are considered as two perpendicular Cartesian coordinates lying horizontally and vertically with positive direction pointing downward can be taken as z-axis. The x-axis is parallel to the direction of propagation of waves. Therefore, the non-zero field of quantities representing the motion is only function of x, z and time t.

The function $I_j(x)$ can be taken as periodic in nature and their Fourier series expansions are provided as (Singh [18]),

$$I_j(x) = \sum_{n=1}^{\infty} \left(I_n^j e^{inkx} + I_{-n}^j e^{-inkx} \right), \; j = 1, 2, \quad j = 1, 2. \tag{1}$$

Here, the Fourier series expansion coefficients are I_n^j and I_{-n}^j, the wave number is k, $\frac{2\pi}{k}$ is the wavelength, n is the order of series expansion and $i = \sqrt{-1}$. We assumed very small amplitude of irregular boundaries compared to the wavelength.

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