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Reflection/refraction of qP/qSV wave in layered self-reinforced media

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ABSTRACT

The work in this paper is concentrated on reflection/refraction phenomena of plane qP/qSV wave through a self-reinforced elastic layer sandwiched between two monoclinic half-spaces. A theoretical approach is given to compute the analytical expressions for reflection and transmission coefficients showing their dependence on relevant parameters. Numerical simulation are performed for a fiber – epoxy resin composites self-reinforced layer sandwiched between Lithium niobate like monoclinic upper and lower strata. Using numerical simulation, dependence of reflection/transmission angles, reflected/transmitted velocities and reflection/transmission coefficient are studies against variation in angle of incidence, normalized wave number and the self-reinforcement parameters for both incident qP and qSV waves. It is found that the resultant deflection angles, velocity and reflection/transmission coefficients are significantly dependent upon elastic coefficients of the media as well as on the self-reinforced parameters. Findings of this paper may help seismologists and geophysicists to enhance knowledge about the rock structures and their elastic properties.

1. Introduction

Seismic waves propagate through the Earth because the material within it, though solid, can undergo internal deformation. As a result, earthquakes and other disturbances generate seismic waves, which give information about both the source of the waves and the material they pass through. These waves in an anisotropic medium are characterized by non-zero flux deviation angles formed by the energy flow direction and wave normal. For any type of anisotropy, there are always three types of waves propagating with three different velocities. Quasi-P, quasi-SH, quasi-SV waves are three mode categories for waves in anisotropic media that exist only in principal material planes. This means that incident and reflected waves in anisotropic media can no longer be thought of as purely longitudinal or shear with appropriate directionally independent wave speeds. This also implies that the direction of energy flow (i.e. group velocity) does not, in general, coincide with the normal to the wave front. An analysis of the reflection–refraction problem in anisotropic elastic media can be found in Knott [1], Musgrave [2], Thapliyal [3], Keith and Crampin [4], Schoenberg [5], Chattopadhyay and Choudhary [6], Chattopadhyay and Rogerson [7]. Anisotropy in the materials buried inside the Earth results from the presence of crystals of particular symmetry or thin laminates. The elastic parameters of crystalline media depend ultimately on chemical composition and atomic arrangement of crystal structure, and hence are characteristic for each mineral. Monoclinic materials possess single plane of

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symmetry. This system corresponds to crystals which can be referred to three not the same axes, two of which intersect at a sloping angle, while the third is perpendicular to the other two. Minerals like lithium niobate, lithium tantalite and orthoclase etc., can be considered as monoclinic materials. Some notable books containing the literature of wave propagation in different elastic media are Brekhovskikh [8], Achenbach [9], Udias [10], Aki and Richards [11] and Nayfeh [12].

In recent years, continued efforts have been expanded upon modeling wave propagation interaction with layered anisotropic media mostly for applications in fields other than seismology, such as nondestructive evaluation. This interest has been prompted by the recent expansion of the use of composite materials in a wide variety of applications like industrial products. A fibre-reinforced composite material with the reinforcement distributed continuously in concentric circles is a material of locally transversely isotropic with the circumferential direction as the preferred direction coinciding with fibre directions. The idea of continuous theory in the fibre reinforced material is developed by Adkins and Rivlin [13], Spencer [14] and Maugin [15] on the theory of large deformations of elastic materials reinforced by inextensible cords. Belfield et al. [16] gave a linear model for fibre-reinforced elastic plates with the reinforcement continuously distributed in concentric circles. A fibre medium in which the body force plays an important role in a uniformed motion of the particles is known as self-reinforced fibre medium. This material is reinforced by strong fibres. In a perfectly conducting fibre-reinforced elastic medium, the motion of the particles creates electric current and hence, a Lorentz body force is generated by the movement of the particles and acts as a body force in the particles motion. This type of medium is a good example of self-reinforced medium. Alumina or concrete in which bridging action develops by gains or aggregates is self-reinforced material. Therefore, seismic wave propagation in a reinforced medium plays an important role in civil engineering and geophysics.

In seismology, the study of wave scattering caused by the Earth's nonuniform internal structure is a subject of interest for acoustics, non-destructive evaluation and mining the oil industry. Several authors contribute their significant work on reflection and refraction problems of seismic waves in different layered anisotropic media. Borcherdt [17] considered the reflection and refraction of general (homogeneous or inhomo-geneous) plane P and SV body waves incident on plane boundaries of general linear viscoelastic solids. He examined the reflection—refraction laws, physical characteristics of the waves, and the nature of critical angles in detail at welded boundaries and a free surface. Borejko [18] discussed the reflection and transmission coefficients for three dimensional plane waves in elastic media. Singh and Khurana [19] presented the closed form algebraic expressions for reflection and transmission coefficients with incident qP or qSV waves at an interface between two monoclinic elastic half space. Reflection/refraction of plane SH wave through a self-reinforced elastic layer between two half-spaces has been discussed by Choudhary et al. [20]. Chattopadhyay and Venkateswarlu [21] discussed the reflection and refraction of plane waves at the interface of fiber reinforced media. Chattopadhyay et al. [22] presented a comprehensive study of reflection and transmission phenomena for three dimensional plane qP-wave incident at an interface of a layered fluid media separated by two distinct triclinic half-spaces. Recently, Kumari et al. [23] studied the reflection and refraction of incident quasi- (p/SV) waves in dissimilar monoclinic media separated with finite isotropic layer.

The work described in this paper is an attempt to discuss the reflection and refraction profile with incident qP/qSV wave in different monoclinic media separated by a self-reinforced layer of finite thickness. The characteristic property of a self-reinforced material is that their components acts together as a single anisotropic unit as long as they remain in elastic condition, i.e., the two components are bound together so that there is no relative displacement between them. Under certain temperature and pressure, some fiber materials may be modified to self-reinforced material. The self-reinforced pressure less sintered silicon carbide bodies exhibit substantially improved fracture toughness as well as flexural strength.

A theoretical approach is given to compute the reflection and transmission coefficients and is compared with the self-reinforcement parameters. The expressions for the phase velocities and amplitude ratios of different reflected and refracted waves have been obtained. Significant deviation in propagation characteristics of incident, reflected and transmitted waves is observed due to self-reinforcement of the intermediate layer, compared to the geometry of intermediate fluid or isotropic layer.

2. Governing equations and formal solutions

The schematic diagram of the problem under consideration is shown in Fig. 1. Here reflection/refraction profile of qP and qSV wave is examined due to incident qP/qSV waves in lower monoclinic substratum H in such a manner that the disturbance is confined at the lower interface of self-reinforced elastic layer $H^{/}$ of thickness h and yields two reflected and refracted qP and qSV waves. Theses refracted waves again impinge on the upper interface of the layer to provide two reflected and refracted waves at $O^{/}$ and $O^{//}$ in upper monoclinic substratum $H^{//}$.

Equations of motion in monoclinic medium with 13 elastic constants in $y-z$ plane (plane of symmetry) are given by

$$C_{66}\frac{\partial^2 u_1}{\partial y^2}+2C_{65}\frac{\partial^2 u_1}{\partial y\partial z}+C_{55}\frac{\partial^2 u_1}{\partial z^2}=\rho\frac{\partial^2 u_1}{\partial t^2}, \tag{1}$$

$$C_{22}\frac{\partial^2 u_2}{\partial y^2}+C_{44}\frac{\partial^2 u_2}{\partial z^2}+C_{24}\frac{\partial^2 u_3}{\partial y^2}+C_{34}\frac{\partial^2 u_3}{\partial z^2}+2C_{24}\frac{\partial^2 u_2}{\partial y\partial z}+(C_{23}+C_{44})\frac{\partial^2 u_3}{\partial y\partial z}=\rho\frac{\partial^2 u_2}{\partial t^2}, \tag{2}$$

$$C_{24}\frac{\partial^2 u_2}{\partial y^2}+C_{34}\frac{\partial^2 u_2}{\partial z^2}+C_{44}\frac{\partial^2 u_3}{\partial y^2}+C_{33}\frac{\partial^2 u_3}{\partial z^2}+2C_{34}\frac{\partial^2 u_3}{\partial y\partial z}+(C_{23}+C_{44})\frac{\partial^2 u_2}{\partial y\partial z}=\rho\frac{\partial^2 u_3}{\partial t^2}, \tag{3}$$

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