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A threshold policy to interrupt transmission of West Nile Virus to birds

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This paper proposes a model of West Nile Virus (WNV) with a Filippov-type control strategy of culling mosquitoes implemented once the number of infected birds exceeds a threshold level. The long-term dynamical behaviour of the proposed non-smooth system is investigated. It is shown that as the threshold value varies, model solutions ultimately approach either one of two endemic equilibria for two subsystems or a pseudo-equilibrium on the switching surface, which is a novel steady state. The results indicate that a previously chosen level of infected birds can be maintained when the threshold policy and other parameters are chosen properly. Numerical studies show that under the threshold policy, strengthening mosquito culling together with protecting bird population is beneficial to curbing the spread of WNV.

1. Introduction

West Nile Virus (WNV), which was first identified in the West Nile subregion of Uganda in 1937, is a mosquito-borne single-stranded RNA virus belonging to the genus Flavivirus in the family Flaviviridae [1–4]. WNV is transmitted between the vector mosquitoes and birds, humans, horses, dogs and other animals, with birds being the most commonly infected animals as well as the principal reservoir hosts [5,6]. Humans and other animals can be infected by the bite of an infectious mosquito that has fed from the blood of an infected bird, but they do not transmit the disease. Thus, WNV is maintained in a mosquito-bird-human transmission cycle in nature [7,8]. WNV has now spread globally and rapidly. The first case in North America was reported in 1999, followed by nearly 40,000 cases and 1554 deaths reported in 52 states up to 2013 [9,10], causing a great deal of concern among the public as well as within federal and State public health and natural resource management agencies.

Owing to the absence of both effective anti-WNV therapeutic treatment for and a vaccine against WNV, it is essential to develop preventive measures or culling strategies to attempt to halt the spread of WNV. It is well known that culling as a tool to control the spread of vector-borne diseases has been extensively used in many studies. For example, in [11], Gourley et al. showed that culling is an effective strategy to control the spread of vector-borne diseases such as WNV and the disease can be eradicated by culling the vector. Tchuenche et al. [12] also investigated the effectiveness of culling strategies on the control of monkeypox transmission while the results indicated that the culling of animals may have counter-productive consequences of increasing cases in children. Numerous mathematical models [4–6,8,13–17] for WNV with cross-infection

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between mosquitoes and birds have been formulated based on Ross-Macdonald equations [18,19], aiming to predict disease dynamics and evaluate possible control strategies. Thomas and Urena [8] formulated a differential equation model to investigate the efficacy of pesticide spraying to reduce mosquito populations and determine how many mosquitoes needed to be killed to ensure elimination of the virus. In [13], a non-spatial SIR model was formulated to examine the emerging WNV epidemic in North America. Bowman et al. [4], proposed a single-season differential equation model in a mosquito-bird-human community (an isolated patch) to assess preventive strategies against WNV. Lewis et al. [17] extended the models formulated in [8,13] by including impulsive events and concluded that a reduction in bird density would exacerbate the epidemic with model in [13], while it would help to maintain the epidemic on the basis of the model in [8]. Blayneh et al. [6] slightly modified the model in [4] to assess the impact of some anti-WNV control measures and obtained the threshold conditions for WNV outbreaks and demonstrated the existence of backward bifurcation in their model. Jiang et al. [16] showed that the dynamics of the whole model in [4] were indeed determined by the four dimensional system involving only the mosquitoes and birds, and suggested that the most effective and realistic strategy to prevent the spread of WNV was to control the mosquitoes. Further, considering impulsive mosquito culling, Hu and Liu et al. [20] formulated a compartmental model in the form of a non-autonomous system of delay differential equations with culling impulses at specific times based on impulsive models [11,21]. Xu et al. [22] formulated two impulsive models considering periodic or state-dependent pesticide sprays as control measures to investigate the transmission of WNV between mosquitoes and birds.

In the continuous models and the models with impulsive control measures at some fixed moments [8,11,13–17,20–22], control is always applied regardless of the population size of infected individuals, which may waste resources because it is not necessary to implement a control strategy when the population density of infected individuals is low. All such models assume, explicitly or implicitly, that interventions are implemented, irrespective of the case numbers and the timing of the implementations. However, differential equation models with state-dependent pulses are proposed to represent strategies that are implemented once the number of infected birds reaches threshold [22]. A common assumption in such models is that the human control activities occur instantaneously, but this is seldom the case with interventions or control strategies usually lasting for a given period. Recently, a threshold policy (TP) has been proposed to describe density-dependent and persistent interventions, which are implemented when the case numbers exceed a certain value and are suspended when they fall below a critical level [23–29]. Therefore, our main purpose is to extend the existing models on WNV as a non-smooth system by considering density-dependent and non-instantaneous control measures, based on the threshold policy idea, to examine whether a threshold policy could be used to control the transmission dynamics of WNV more effectively than reliance on existing impulsive differential equations. We then aim to identify the most rational threshold or most effective strategies to control the transmission of WNV and keep the number of infected birds relatively low.

To achieve the above goals, we formulate a non-smooth model with a Filippov-type control strategy by assuming that control interventions are implemented once the number of infected birds exceeds a certain level, investigate the transmission of WNV between mosquitoes and birds theoretically, and draw some interesting conclusions. The paper is organised as follows. In Section 2, an epidemic model with a Filippov-type control is proposed to describe the non-instantaneous control and the dynamics of two subsystems are analysed. In Section 3, sliding mode dynamics and the existence of the pseudo-equilibrium are investigated. The boundary node bifurcation is discussed in Section 4 and the global behaviour of the system is described in Section 5. Finally, we provide biological conclusions and discussion in Section 6.

2. Epidemic model of WNV transmission with a Filippov-type control and preliminaries

Under threshold policy (TP), control is implemented when the case number exceeds a critical level, while it is suppressed when it is below the specific threshold level. We assume that mosquitoes and birds are culled when the number of infected birds in a population exceeds a certain level E_{I_b}. Let $N_m(t)$ represent the total female mosquito population at time t, divided into two classes: uninfected susceptible female mosquitoes ($S_m(t)$) and female mosquitoes infected with WNV ($I_m(t)$) (i.e. $N_m(t) = S_m(t) + I_m(t)$). Similarly, for the birds, $N_b(t)$ denotes the total bird population at time t and $N_b(t) = S_b(t) + I_b(t)$ in which $S_b(t)$ is the number of susceptible birds and $I_b(t)$ stands for birds infected with WNV. The recruitment rate and natural death rate are Λ and μ, for which we use the subscript m and b to identify the female mosquitoes and birds. The susceptible mosquitoes enter into the infected mosquitoes category when they bite infected birds, at an average biting rate c and with a probability β_{mb} for transmission of WNV from birds to mosquitoes. Similarly, susceptible birds join the infected birds class when they are bitten by infected mosquitoes at an average biting rate c and with a probability β_{bm} for transmission of WNV from mosquitoes to birds. f_m and f_b denote the culling rates of mosquitoes and birds, respectively. The model variables and definitions of the parameters are listed in Table 1.

$$\begin{cases} \dfrac{dS_m}{dt} = \Lambda_m - c\beta_{mb}\dfrac{I_b}{N_b}S_m - \mu_m S_m - \epsilon f_m S_m, \\ \dfrac{dI_m}{dt} = c\beta_{mb}\dfrac{I_b}{N_b}S_m - \mu_m I_m - \epsilon f_m I_m, \\ \dfrac{dS_b}{dt} = \Lambda_b - c\beta_{bm}\dfrac{S_b}{N_b}I_m - \mu_b S_b - \epsilon f_b S_b, \\ \dfrac{dI_b}{dt} = c\beta_{bm}\dfrac{S_b}{N_b}I_m - \mu_b I_b - \epsilon f_b I_b, \end{cases} \tag{1}$$