Applied Mathematical Modelling 000 (2016) 1–11

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Joint replenishment model for multiple products with substitution

Bacel Maddah[a], Mohamed Kharbeche[b], Shaligram Pokharel[c,*], Ahmed Ghoniem[d]

[a] Department of Industrial Engineering and Management, Faculty of Engineering and Architecture, American University of Beirut, Beirut 1107 2020, Lebanon
[b] Qatar Road Safety Studies Center, College of Engineering, Qatar University, Doha, Qatar
[c] Department of Mechanical and Industrial Engineering, College of Engineering, Qatar University, Doha, Qatar
[d] Operations & Information Management Department, Isenberg School of Management, University of Massachusetts Amherst, 121 Presidents Dr., Amherst, MA 01003, USA

A R T I C L E I N F O

Article history:
Received 12 March 2015
Revised 2 February 2016
Accepted 24 March 2016
Available online xxx

Keywords:
Product substitution
Inventory control
Joint replenishment

A B S T R A C T

We extend the classical economic order quantity model to address the joint replenishment of multiple products under substitution. The proposed model optimizes ordering quantities for each product under substitution effects with the objective of minimizing the total cost associated with the setup, holding, and shortage of products, while partially meeting demand. First, the special case of three substitutable products is examined in detail. Then, a nonlinear mathematical programming formulation is presented as a general-purpose solution approach for any number of substitutable products. The convexity of the model is discussed. We find that the objective function to be convex in the important special case of products having equal unit holding costs, which typically holds for substitutable products in practice. Sensitivity analysis is conducted in order to determine the impact of cost parameters variations on the ordering policy. We focus on identifying conditions that favor substitution among products. We find that allowing substitution among products is an effective vehicle for cost cutting in supply chain settings involving high fixed costs, low holding costs, and low shortage costs.

1. Introduction

Joint replenishment of products is a common approach in supply chain management, which is sought to reduce fixed costs, e.g., by shipping several products on one truck. Joint replenishment may be also needed for practical logistical purposes. For example, many retailers prefer to receive certain types of goods (e.g., groceries) at a specific time of the day or the week. The popularity of joint replenishment practices is reflected by a wide academic research on the topic as indicated

* Corresponding author.
E-mail addresses: bacel.maddah@aub.edu.lb (B. Maddah), mkharbec@qu.edu.qa (M. Kharbeche), shaligram@qu.edu.qa (S. Pokharel), aghoniem@isenberg.umass.edu (A. Ghoniem).

http://dx.doi.org/10.1016/j.apm.2016.03.027

in the review paper of Khouja and Goyal [1]. Examples of recent works on joint replenishment include Hong and Kim [2], Porras and Dekker [3], Schulz and Telha [4], Silva and Gao [5] Zhang et al. [6], and Zhou et al. [7].

Demand substitutability commonly occurs for products that are similar in nature (e.g., different brands of coffee or soda) and may be jointly ordered/shipped. Under such settings, customers of one product can switch to another similar product due to price or availability. Analyzing the effect of demand substitution on the inventory management of several related products is also an important problem that has received wide attention in the literature. Detailed accounts of the demand substitution literature can be found in the review papers by Kök et al. [8], Mahajan and van Ryzin [9], and Maddah et al. [10]. Recent literature has focused on stochastic demand substitution driven by consumer choice models adapted from the economics and marketing literature (e.g., van Ryzin and Mahajan [11], Mahajan and van Ryzin [12], Smith and Agrawal [13], Cachon and Kök [14], Gaur and Honhon [15], Gurler and Yilmaz [16], Maddah and Bish [17], and Maddah et al. [10]).

Despite the relevance of the joint replenishment practice to substitutable products, the academic literature has devoted little attention to studying the joint replenishment of substitutable products. Notable exceptions include the studies by Drezner et al. [18], Gurnani and Drezner [19], and the recent work by Salameh et al. [20]. Drezner et al. [18] study the joint replenishment of two substitutable products having deterministic demand in an economic order quantity setting, assuming a one-to-one substitution, where the demand of a product is fully substituted by another product in the event of a stock-out of the first product. This type of substitution is possible in manufacturing settings. Gurnani and Drezner [19] extend the work of Drezner et al. [18] to analyze joint replenishment of multiple, two or more, products. Gurnani and Drezner [19] also assumed a one-to-one substitution and consider a type of one-way substitution where customers could “upgrade” to a set of higher quality products in the event that their most preferred product is stocked out.

Recently, Salameh et al. [20] consider a two-product joint replenishment model with substitution in an EOQ framework similar to Drezner et al. [18]. However, they allowed for partial substitution, meaning that in the event of a product stock-out, a fraction of its customers will substitute to the other product, while the remainder customers will chose not to buy, leading to loss sales, which incurs a penalty. Salameh et al. [20] also allowed for a two-way substitution. They suggest adopting the substitution direction which has the lowest cost, by solving two related problems with (i) the second product substituting the first, and (ii) vice versa. Krommyda et al. [21] consider a problem similar to Salameh et al. [20] of two products under two-way, stock-out, and partial substitution within the EOQ framework; but further assume that the demand of a product is stimulated by the inventory levels of *both* products, in an interesting extension of the single-product literature with stock-dependent demand.

In this paper, we consider the joint replenishment of multiple, three or more, substitutable products, in an EOQ framework, under a versatile substitution model where every product in-stock can partially substitute a stocked-out product. In addition, a fraction of customers may elect not to substitute their most preferred product, and a lost sales penalty is charged. We first develop our substitution model and cast it into a nonlinear programming model, and then draw useful managerial insights. Our work can be seen as an extension of the two-product work of Salameh et al. [20] to the more challenging case of three or more products. We differ from the work of Gurnani and Drezner [19] in that we consider partial two-way substitutions, while they consider one-to-one, one-way substitution. Moreover, our mathematical model is more general than that of Gurnani and Drezner [19], which can be seen as a special case of our model. More notably, our work is applicable to retailing, while that of Gurnani and Drezner [19] is more adequate for manufacturing contexts.

At this point, it is worth clarifying what is exactly meant by “substitutable products” in this paper. For our purpose in this paper, the substitutable products we consider belong to a set of products that serves the same basic need for the consumer (e.g., drinking coffee, brushing teeth, or washing clothes.) but differ in some secondary aspect such as color, flavor, or smell. This is, for example, the case of several fast-moving consumer goods (FMCG) categories that are offered by super markets, e.g., coffee, toothpaste, and washing detergent. The economics literature refers to this type of substitutable products as “horizontally differentiated”, and generally considers such products to have equal or approximately equal unit costs and different demand rates (e.g., Anderson et al. [22]).

Finally, it is worth commenting on the EOQ setting utilized in this paper. The EOQ model is among the most popular inventory systems, especially in academic studies (see, for example, Silver et al. [23] and Zipkin [24] for overviews). While some authors defend the applicability of the EOQ model in practice (e.g., Osteryoung et al. [25] and Silver [26]), many criticize its applicability, mainly due to the difficulty is estimating its related costs (e.g., Jones [27], Selen and Wood [28], and Sprague and Sardy [29]). Jaber et al. [30] attempt to rectify the limitations of the EOQ model by appealing to thermodynamics principles with an “entropy cost” capturing hidden costs. Subsequently, several extensions of the base model in Jaber et al. [30] took place to account for different complicating factors such as delays in payments, deterioration effects, and supply chain effects, among other things, as nicely summarized in Jaber [31]. It is worth noting than none of the works surveyed in Jaber [31] considers product substitution effects such as the ones mentioned in this paper, which could be an interesting area for future work, especially that it might be difficult to estimate the substitution rates and the stock-out costs of our model in some settings.

The remainder of this paper is organized as follows. Section 2 provides a formal problem statement and our model for the case of three substitutable products. Section 3 extends the model to $n \geq 4$ products, presents the solution methodology in order to get the optimal ordering quantities, and establishes convexity properties of the cost function. In Section 4, a numerical study is presented which leads to several managerial insights. Finally, in Section 5, we conclude the paper with a summary of our findings and directions for future research.