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Mathematical modeling and sensitivity analysis of arterial anastomosis in the arm

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ABSTRACT

Mathematical modeling is a powerful tool for predicting and investigating cardiovascular (CV) diseases. It has been shown that the lumped-parameter model can draw on an analogy between electrical circuits and fluid flow to provide a simple but effective method for modeling the human cardiovascular system.

In order to build a patient-specific CV model, it is not practically feasible to estimate numerous model parameters (electrical and structural) from patient-specific data (measurements). However, estimation may be possible if optimal measurement locations are identified for important model parameters with complementary optimal time regions. Thus, sensitivity analysis can be used to identify important parameters as well as the optimal measurement locations and optimal time regions in the pressure and flow waves.

In this study, we applied parametric local sensitivity analysis to a linear elastic lumped-parameter model of the arm arteries (with and without anastomosis) to identify important electrical and structural parameters. The ultimate goal of this study is to provide guidance for experimentalists about what to measure (pressure and flow) and where (network locations) in order to estimate the key CV model parameters, which are the first steps required to build a patient-specific CV model.

1. Author summary

Cardiovascular (CV) diseases are among the leading causes of death throughout the world and the number of patients is increasing worldwide. Reliable model predictions are required to investigate CV diseases during the early stages. In computational CV models, the parameters are major sources of uncertainty, which make these models unreliable and their predictions are not accurate. In order to obtain predictive models that facilitate investigations of CV diseases, sensitivity analysis can be used to quantify and reduce the output uncertainty caused by the model parameters. In this study, we developed a mathematical model of the arm artery (with and without arterial anastomosis) where we applied local sensitivity analysis to identify the optimal measurement locations and optimal time regions in the pressure and flow waves to estimate the key CV parameters. Thus, better estimates of the key parameters can be expected if we only consider the optimal measurement locations as well as the optimal time regions for pressure and flow waves. The results demonstrated that the sensitivity of

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the pressure and flow to each model parameter depended greatly on the locations considered on arm arteries. Strong pressure and flow sensitivities in the brachial artery were observed with respect to compliance, blood inertia, and the diameter and the length of the vessel, whereas Young's modulus and wall thickness were found to be less influential parameters.

2. Introduction

Various mathematical models have been developed and applied due to growing interest in the prediction and diagnosis of cardiovascular (CV) diseases. Lumped-parameter models (based on an electrical analogy to fluid flow) are known to be effective for modeling the human CV system [1–10]. A mathematical model is a simplified version of a real-world problem, so the reliability of the CV models used for medical decision making depends upon the accuracy of the model outputs. The model outputs rely mainly on the input factors, i.e., the parameters and their feasible regions, input and output boundary conditions, model structure, and spatiotemporal variability. In practice, the values of all the input factors are not known precisely, which introduces further uncertainty into the outputs due to the imprecise input factors.

In order to make the predictions of CV models more accurate to explain the most important features of the real process, the output uncertainty can be reduced by sensitivity analysis. During sensitivity analysis, the uncertainty in the outputs of a mathematical model or system can be related to different sources of uncertainty in the inputs [11–19]. Sensitivity analysis is a powerful approach for identifying important CV system parameters [20–23]. In order to reduce the output uncertainty, the important parameters can be estimated further using clinically obtained measurements [24,25].

The simplest and most efficient form of the sensitivity analysis is to vary one model parameter at a time by a given amount and examine the impact on the output results. The analysis can be repeated for all of the model parameters independently at different times. The method is known as local sensitivity analysis (LSA) or one-factor-at-a-time [26–28].

Before performing a sensitivity analysis, it is important to know the input and output quantities of interest. In the present study, the input quantities of interest are the electrical and structural parameters, whereas pressure and flow at all locations on the arm arteries are considered as the output quantities of interest.

In this study, we aimed to determined how structural changes such as anastomosis influence local sensitivity. Anastomoses are interconnections between vessels, which provide a collateral circulation as well as acting as a secondary route for blood flow when the main vessels are blocked by plaque, atherosclerosis, or stenosis to minimize damage at the tissue level. However, we did not consider the turbulent effects that appear in merging flows at the end-to-side anastomosis, which depend mainly upon the angle and flow rate of the merging vessels.

In the present study, we developed a lumped-parameter model of anastomosis around the elbow joint (superior ulnar collateral anastomosis with posterior ulnar recurrent; SUC-PUR). The luminal diameter (or equivalently, the flow resistance) is an important parameter, so we concentrated on the sensitivity of the pressure and flow with respect to the viscous flow resistance and boundary resistance.

Considering the geometry of the arm arteries (with and without anastomosis) with input and output quantities of interest, the aims of study are summarized as follows.

(1) To identify the CV parameters with the greatest effects on the hemodynamic state variables: pressure and flow.
(2) To identify the optimal measurement locations and optimal time regions in the flow and pressure waves w.r.t. each CV parameter. This information can be used to set the optimal measurement locations for state variables to estimate the most influential CV parameters.
(3) To discuss and explain the impacts of electrical parameters in arterial anastomosis.

In this study, we also aimed to validate different LSA methods by using a simple example problem of an arm artery to demonstrate the agreement with the principle of Ohm's law of hydrodynamics. However, this method could be used to analyze more complex CV networks.

3. Network structure and model equations

The domain decomposition approach is used to generate the model of arm arteries (with and without anastomosis), where the arm arteries are decomposed into a number of vascular segments and the parameters are approximately constant. Each non-terminal and terminal segment of the arm arteries in a network structure is represented by its corresponding non-terminal and terminal electrical circuit, as shown in Fig. 1.

However, decomposition into vascular segments requires the relationships between the arterial segments to reconstruct the network structure of the arterial tree. Therefore, bifurcation conditions are defined for the mother and daughter vessels as follows:

$$q_1 = q_2 + q_3, \quad \text{(diverging)} \tag{1}$$

$$q_3 = q_1 + q_2, \quad \text{(merging)} \tag{2}$$

$$p_1 = p_2 = p_3. \tag{3}$$

These conditions are derived from the conservation of mass and momentum, i.e., pressure is constant and flow must be conserved at the bifurcation (see Fig. 2). For further details, please refer to [8].

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