Applied Mathematical Modelling 000 (2016) 1–12

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A priority based assignment problem

Prabhjot Kaur[a], Anuj Sharma[c], Vanita Verma[b], Kalpana Dahiya[a,*]

[a] University Institute of Engineering and Technology, Panjab University, Chandigarh 160014, India
[b] Department of Mathematics, Panjab University, Chandigarh 160014, India
[c] Department of Computer Science and Applications, Panjab University, Chandigarh 160014, India

ARTICLE INFO

Article history:
Received 8 April 2015
Revised 25 January 2016
Accepted 29 March 2016
Available online xxx

Keywords:
Optimization
Non-linear programming
Combinatorial analysis
Allocation
Assignment problem

ABSTRACT

This paper discusses a priority based assignment problem related to an industrial project consisting of a total of n jobs. Depending upon its work breakdown structure, the execution of the project is carried out in two stages where the m primary jobs are performed first, in Stage-I whereas the $(n-m)$ secondary jobs are performed later in Stage-II (as the secondary jobs cannot be performed until the primary jobs are finished). A number of manufacturing units exactly equal to n, each of them capable of performing all the n jobs involved in the project, are available. A tentative job-performance time taken by each of these manufacturing units for each of the n jobs is available. The purpose of the current study is to assign the jobs to the manufacturing units in such a way that the two-stage execution of the project can be carried out in the minimum possible time. For this, a polynomial time iterative algorithm is proposed, which at each iteration, aims at selecting m manufacturing units to perform primary jobs corresponding to which, the remaining $(n-m)$ manufacturing units perform the secondary jobs optimally and from this selection, a pair of times of Stage-I and Stage-II is obtained. The proposed algorithm is such that at each iteration, time of Stage-I decreases strictly and time of Stage-II increases. Out of the pairs so generated, the one with minimum sum of Stage-I and Stage-II times is considered as optimum and the corresponding assignment as the optimal assignment. A numerical illustration is given in the support of the theory. Also, the proposed algorithm is implemented and tested on a variety of test problems and the average run time for each problem is calculated.

1. Introduction

The assignment problem is one of the fundamental combinatorial optimization problem in the branch of optimization. In an assignment problem, one is interested in finding the cheapest possible way of assigning n jobs to the n agents such that each agent is assigned to a unique job and each job is performed by one and only one agent. It is also assumed that all agents perform in parallel. If $I = \{1, 2, \ldots, n\}$ is the index set of n agents, $J = \{1, 2, \ldots, n\}$ is the index set of n jobs and c_{ij}, $\forall\ (i, j) \in I \times J$ is the cost incurred in assigning the jth job to the ith agent, then a cost minimization assignment problem (CMAP) can be stated as:

$$\min_{X=\{x_{ij}\}\in S} \sum_{i\in I} \sum_{j\in J} c_{ij}x_{ij}$$

* Corresponding author. Tel.: 0172-2534515
E-mail addresses: prabh.g3@gmail.com (P. Kaur), anujs@pu.ac.in (A. Sharma), v_verma1@yahoo.com (V. Verma), kalpanas@pu.ac.in, kalpana_math@yahoo.co.in (K. Dahiya).

http://dx.doi.org/10.1016/j.apm.2016.03.038
S0307-904X(16)30186-X/

where S is defined as

$$S = \left\{ X = \{x_{ij}\} \in R^{n^2} \;\middle|\; \begin{array}{l} \sum_{j\in J} x_{ij} = 1, \quad \forall\, i \in I \\ \sum_{i\in I} x_{ij} = 1, \quad \forall\, j \in J \\ x_{ij} = 0 \text{ or } 1, \quad \forall\, i \in I,\ j \in J \end{array} \right\}$$

where $x_{ij} = 1$, if ith agent performs jth job and 0 otherwise. There exist various algorithms for CMAP, sequential and parallel, ranging from primal-dual combinatorial algorithms to simplex-like methods. The first algorithm for CMAP was presented in 1946 by Easterfield [1] which was not a polynomial time algorithm. The first polynomial time primal-dual algorithm for CMAP is the Hungarian method. This method was proposed in the mid 1950s by Kuhn [2] and its original formulation solves the problem in $O(n^4)$ time. Later on, it was proved that the best time complexity for Hungarian algorithm is $O(n^3)$ [3]. There are number of survey papers and books on algorithms available to solve CMAP. For the various results on CMAP proved in 1980s and 1990s, see annotated bibliography of Dell'Amico and Martello [4].

In the recent years also, many researchers have studied variants of assignment problems and proposed different methods to solve them [5,6]. Assignment problems under fuzzy environment also attracted the interest of many researchers. Some of these studies are given in [7–9]. Krumke and Thielen [10] studied generalized assignment problem with minimum quantities. A branch-and-price approach for the generalized assignment problem is given by Sarin et al. [11].

In most real world problems, especially in business world, the complexity of the social and economic environment requires the explicit consideration of objective function other than cost. One of the most important factor that may flourish or affect a business's prosperity is the time. The business coaches often focus on time management tips with their clients, because the business owner's personal effectiveness directly impacts the business. Therefore, in-time completion of a project becomes one of the most important challenges for an organization. Many researchers have worked upon the problems related to project scheduling with the objective of completing the project in minimum time (see [12,13]).

Consider an industry manufacturing a product which requires some agents to do the jobs involved in the manufacturing process and according to the work efficiency of each of these agents, there is a time associated with each agent-job link. The objective is to assign the jobs to the agents in such a way that minimum overall completion time of the project is obtained. This gives rise to an important class of assignment problems viz. time minimization assignment problem (TMAP). Mathematically, if t_{ij} is the time taken by the ith agent to complete jth job, then TMAP, also known as bottleneck assignment problem can be formulated as:

$$\min_{X\in S} \left[\max_{I\times J} (t_{ij}(x_{ij})) = T(X) \right]$$

where

$$t_{ij}(x_{ij}) = \begin{cases} t_{ij}(\geq 0) & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} = 0. \end{cases}$$

over the same set of constraints as defined for CMAP above. This kind of problem was first studied by Fulkerson et al. [14] in 1953. Important studies in TMAP have been made by Garfinkel [15], Ravindran [16], Carpento and Toth [17], Derigs [18] and many other researchers. Burkard and Franz [19] studied the lexicographic bottleneck problems. The algorithm to compute weights for lexicographic optimal solutions was proposed by Sherali [20] in 1982. Mazzola and Neebee [21] have developed an algorithm for bottleneck generalized assignment problem. Many other research papers can also be found in which other aspects of this kind of a problem are explored.

In 1980, Bansal and Puri [22] proved that $T(X)$ is a concave function. Thus TMAP involves minimization of a concave function over an assignment polytope and hence it belongs to the class of concave minimization problems (CMP). As the minimizer of a CMP over a polytope is attainable at its extreme point, the search for an optimal solution is restricted to the set of extreme points only. Almost all the techniques for solving TMAP involve an ordinary CMAP for which strongly polynomial algorithms are known to exist (see [23,24]). Therefore, it follows that a TMAP is also solvable in strongly polynomial time. The best time complexity for TMAP is $O(n^2\sqrt{n\ logn})$ [25].

The heavy commercial vehicles (HCVs) form an indispensable part of the automobile industry all over the world. Over the years, global market has witnessed the launch of many HCVs from different manufacturers and the requirement and demand for HCVs is escalating with every passing day considering the growing pace of the fiscal expansion of a nation. For an industry, there is need of a strategy that can allow its organization to concentrate its resources on the optimal opportunities with the goals of increasing sales and achieving a sustainable competitive advantage. Developing such a marketing strategy is vital for any business. Without one, its efforts to attract customers are likely to be haphazard and inefficient. One of the important part of such a strategy, which may have a crucial influence on the business, is to deliver its product to the customers in time. So, all the above factors which have direct impact on the business motivated us to study an automobile industrial project of constructing heavy commercial vehicles HCVs.

In most of the industrial projects, there is a work breakdown structure (WBS) which provides a deliverable-oriented decomposition of the project into smaller components in which some of the components must be performed by the agents before performing the others. Therefore, in such a project, there are some jobs (called primary jobs) which are supposed