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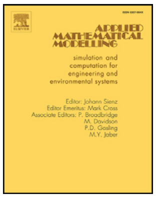

Impact of wind power generation on a large scale power system using stochastic linear stability

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ABSTRACT

The effect of random and sustained disturbances is studied in this paper. The Ornstein–Uhlenbeck stochastic process is proposed with the aim to represent the variations in the power output produced by wind power generation, considering measurements of a wind farm located in the fourth region of Chile as real field data for its parameters calibration. An example is provided considering the study of the exponential stability using the Lyapunov exponent as an indicator, with the aim to determine the maximum size of a wind farm that can be connected in a busbar of one of Chile's interconnected power system (Sistema Interconectado del Norte Grande, SING), considering successive increments from the output power from the connected wind farm.

1. Introduction

Dynamic and permanent regime studies that are made on electric power systems (EPS) are of vital importance to the electric industry, because they make it possible to determine the adequate operating conditions for supplying the electric power required by society in an economic, reliable and safe manner. In this context, the most important approaches of the EPS studies are oriented at their planning and operation. One of the main problems that concern these topics consists in keeping the system operating in a steady state, i.e., that the system does not lose its balance when it is subjected to perturbations that affect its behavior.

In their normal operation, electric power systems are subjected to a wide variety of random and sustained in time disturbances, which affect all aspects of the real-time operation of the system. The most common examples of random disturbances affecting the interconnected systems are, e.g., variations in the distribution system by the fluctuation in the electrical demand of the customers and random changes in generation that depend of some non-controllable energy sources. In that aspect, electrical generation that uses wind energy as its main source brings itself random variations to the power systems that depend of the statistical characteristics of the wind speed at its location.

Within the context of transient stability, the random behavior of the system has been approached considering different system scenarios and parameters which are associated with an occurrence probability [1–9]. In terms of quantitative

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assessment, [10] presents an index that allows determining the vulnerability when facing a voltage collapse, establishing that consumption has a random behavior. This stability index corresponds to the time for leaving a stable operation zone.

Stability studies of small probabilistic disturbances are proposed in [11–19]. One of the main approaches is to assign a probability distribution of the real parts of the eigenvalues obtained from the linear equivalent model of the electric system, and then determine the probability that the real parts will be located in the left half-plane. In this context, applications are also presented in which the PSS (Power Systems Stabilizer) controllers are used to decrease the effect of the disturbances that affect the operation but occur in a single instant of time.

To account for the random and permanent effect over time, Lyapunov exponent have been used as stability indices to analyze power systems [20]. Reference [21] gives a theoretical description of the calculation of Lyapunov exponents in structures, considering low dimension systems as applications. However, these studies are still in a theoretical stage and no numerical methods for estimating them are shown.

With the purpose of characterizing faithfully the presence of random perturbations self-sustained over time, it is necessary to consider stochastic models that account for the actual dynamics that take place during the operation of the system. This fact shows the need to use stochastic differential equations to describe completely what happens in the system.

Considering the above, the purpose of this paper is to model stochastically the behavior of certain components of the EPS. In particular, a representation will be made of the generating power of a wind farm by means of a model that accounts for the random and self-sustained over time dynamics.

The treatments found in the specialized literature with respect to the kinds of stochastic models for the EPS are summarized in what follows. Reference [10] includes Brownian motion in the system's dynamic model to describe the load component, in order to study the vulnerability of the system to voltage collapse.

In [22] SDE are used as planning tools in power systems, using a stationary Gaussian process with constant spectral density to model the random variations in lines and loads. In [23] they are used to model small perturbations in load systems and transmission line parameters. In [24] they are applied to analyze the dynamics of an EPS, including discrete perturbations produced by the tap changing operation in a transformer. In references [22,25,26] they are used for stability studies, modeling the behavior of the loads by means of the Ornstein–Uhlenbeck (O-U) process and Ito's nonlinear differential equation. Reference [27] models the wind generation and the loads by the O-U process, but it does not present a validity test of the estimated parameters that can ensure a good fit of the model.

In brief, the specialized literature shows important progress in the study of power systems stability considering random self-sustained disturbances and the Lyapunov exponent as an index of the stability behavior of the system. However, considering the previously stated, it is still necessary to study this topic for large scale power systems, and also, the modeling of the random phenomena taking into account the calibration of the stochastic process using real field measurements. This paper advances in both areas, firstly, the mathematical treatment of the random stochastic disturbance is done considering measurement of an actual wind power park, and the Lyapunov exponent is studied for the injection of wind parks in a real large scale power system (SING).

This paper is organized as follows. Section 2 shows the methodology for the analysis of electrical power systems under stochastic disturbances, considering the Lyapunov exponent concept as an stability index for stochastic linear systems, and the mathematical model of the random perturbation, considering real field measurements for the modeling. In Section 3 the application case is presented, where the characterization of random disturbances from wind power plants using the Ornstein–Uhlenbeck process is shown; an example study case is provided, showing the impact of large scale wind power injection in the SING, considering the analysis of the stability of the interconnected system following stochastic disturbances. This analysis is done taking into account the integration of a wind power plant in one of the busbars of the interconnected system and considering a successive increases in the size of the disturbance, such that the Lyapunov exponent can be regarded as an indicator of the global stability of the system. Finally, in Section 4, conclusions and future work are discussed.

2. Methodology

In this section we describe the general methodology to assess the influence of random disturbances from wind power generation on the steady-state performance of a large scale power system, in this case the SING.

2.1. Lyapunov exponent

To study the stability of power systems subjected to stochastic disturbances, a multiplicative model perturbation is considered. This representation allows us to use the Lyapunov Exponent concept as a stability index for the analysis of the steady-state operation of power systems with random variability. In first place, consider the following non-linear equation system:

$$\dot{y} = f(y, p). \tag{1}$$

The system is defined in the state space $\mathbb{R}^N$, where $N_1 = 2n$ corresponds to the relative angles of the rotors $(\delta_1, \ldots, \delta_n)$ and velocities $(\omega_1, \ldots, \omega_n)$ and N_2 corresponds to the rest of the state variables $(N_1 + N_2 = N)$. The vector $p \in \mathbb{R}$ corresponds to the variables that can be fitted to guarantee the optimum operation of the system.

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