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On the irreversibility of Moore cellular automata over the ternary field and image application

Selman Uguz[a,*], Hasan Akin[b], Irfan Siap[c], Ugur Sahin[d]

[a] Department of Mathematics, Arts and Sciences Faculty Harran University, Şanliurfa 63120, Turkey
[b] Hasan Akin, Department of Mathematics, Faculty of Education, Zirve University, Gaziantep 27260, Turkey
[c] Irfan SIAP, Department of Mathematics, Arts and Science Faculty, Yıldız Technical University, Istanbul, Turkey
[d] Ugur Sahin, Rochester Institute of Technology, Multi Agent Biorobotic Laboratory, Rochester, NY, USA

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ABSTRACT

Cellular automata have rich computational properties and provide many models in mathematical and physical processes. In this paper, one of the most commonly used neighborhood types of two dimensional (2D) cellular automata which is called the Moore neighborhood in two dimensional integer lattice is considered. We study the characterization of 2D linear cellular automata defined by the Moore neighborhood with periodic and null boundary conditions over ternary fields. Furthermore, we analyze some results of 2D cellular automata defined by the rule number *9840NB* and finally also present applications to error correcting codes and image processing field.

1. Introduction

A cellular automaton (plural cellular automata, abbreviated to CA) is a discrete model studied and applied in many areas of science. Cellular automata (CAs) have rich computational properties and provide different amazing models in computation. CAs were first used for modeling various physical and biological processes and especially in computer science. The study of CAs has received remarkable attention in the last few years [1–7], because CAs have been widely investigated in many disciplines with different purposes (e.g., simulation of natural phenomena, pseudo-random number generation, image processing, analysis of universal model of computations, coding theory, cryptography).

CAs provide a good source in parallel processing which is used in coding, cryptography and network applications in general. For instance, CAs computations with secret key is discussed in [8]. CA is also used to design a symmetric key cryptography system based on Vernam cipher [8]. Furthermore, CA (especially hybrid CA) is used in generating pseudo-random numbers sequence (PNS) which are applied in the encryption processes. The quality of PNSs highly depends on the set of applied CA rules and the system is very resistant to attempts of breaking the cryptography key.

Most of the studies and applications for CA is done for one-dimensional (1D) CA. “The Game of Life” developed by John H. Conway in the 1960s is an example of a two-dimensional (2D) CA. John von Neumann in the late 40’s and early 50’s studied CA singular [9]. 2D CA with von Neumann neighborhood has found many applications and been explored in the literature [10,11]. Recently, 2D CA have attracted much of the interest [7,10,12,13]. Some basic and precise mathematical

* Corresponding author. Tel.: +90 5322217013.
E-mail addresses: selmanuguz@gmail.com (S. Uguz), hasanakin69@gmail.com (H. Akin), isiap@yildiz.edu.tr (I. Siap), useee@rit.edu (U. Sahin).

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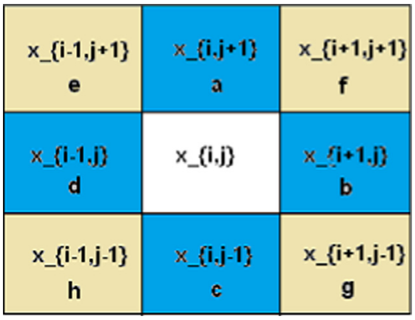

Fig. 1. 8 elements of the Moore neighborhood surround the center x_{ij}.

models using matrix algebra built on field $\mathbb{Z}_2$ were reported for characterizing the behavior of two-dimensional nearest neighborhood linear CAs with null or periodic boundary conditions [2–4,6,11,14,15].

In this paper, we investigate the characterization of 2D finite linear CA (shortly, 2D FLCA) with the Moore neighborhood by using matrix algebra built on $\mathbb{Z}_3 = \{0, 1, 2\}$ (The set of integers modulo 3). We present and study 2D linear CA with a new local rule called rule 9840 over ternary fields. This local rule is one of the important rules since it includes all possible neighbors of radius one. Here, we analyze some results about the rule numbers 9840NB (null boundary) and 9840PB (periodic boundary). We construct the rule matrix corresponding to the Moore CA. In order to compute the rank of the rule matrix the corresponding 2D finite Moore CA, by applying elementary row and column operations to the matrix, we obtain recurrences equations. In the present paper, the properties of two dimensional rules 9840NB and 9840PB over ternary fields are investigated and the dimension of the kernel is established via some recurrence relations. We examine whether 2D FLCA with the Moore neighborhood is reversible. Finally, we present two applications of this family to error correcting codes by extending the original method from binary fields to ternary fields and image processing area.

The present paper is organized as follows. Section 2 present basic technical details about 2D Moore CA over the ternary field $\mathbb{Z}_3$. Reversibility of mathematical model for Moore CA with null boundary case (Rule 9840NB) is given Section 3. In Section 4, an application of error correcting codes of CA is given in details. Application of 2D linear Moore CA in image processing (for the rules 9840NB, 9840PB) is presented in Section 5. Finally conclusions of the present study are briefly summarized in Section 6.

2. Two-dimensional CA over the field $\mathbb{Z}_3$

The 2D finite CA consists of $m \times n$ cells arranged in m rows and n columns, where each cell takes one of the values of the field $\mathbb{Z}_3$. From now on, we will denote 2D finite CA order to $m \times n$ by 2D CA$_{m \times n}$. A configuration of the system is an assignment of the states to all cells. Every configuration determines a next configuration via a linear transition rule that is local in the sense that the state of a cell at time $(t+1)$ depends only on the states of some of its neighbors at the time t using modulo 3 algebra. For 2D CA nearest neighbors, there are nine cells arranged in a 3×3 matrix centering that particular cell (see [2] for details).

2.1. Moore neighborhood

In 2D CAs theory, there are some classic types of neighborhoods, but in this paper we only restrict ourselves to the special neighbors which is called Moore neighborhood. The Moore neighborhood comprises the eight cells surrounding the central cell on a two-dimensional square lattice. It is one of the two most commonly used neighborhood types, the other one is the 4-cell von Neumann neighborhood (see [11]). The Moore neighborhood was used in the well known Conway's Game of Life. It is similar to the notion of 8-connected pixels in computer graphics [12,13,15].

In Fig. 1, we show the Moore neighborhood which comprises eight square cells which surround the center cell $x_{(i,j)}$. The state $x_{(i,j)}^{(t+1)}$ of the cell (i, j)th at time $(t+1)$ is defined by the local rule function $f : (\mathbb{Z}_3)^9 \to \mathbb{Z}_3$ as follows:

$$\begin{aligned} x_{(i,j)}^{(t+1)} &= f\left(x_{(i,j)}^{(t)}, x_{(i+1,j)}^{(t)}, x_{(i+1,j-1)}^{(t)}, x_{(i,j-1)}^{(t)}, x_{(i-1,j-1)}^{(t)}, x_{(i-1,j)}^{(t)}, x_{(i-1,j+1)}^{(t)}, x_{(i,j+1)}^{(t)}, x_{(i+1,j+1)}^{(t)}\right) \\ &= a_0 x_{(i,j)}^{(t)} + a_1 x_{(i+1,j)}^{(t)} + a_2 x_{(i+1,j-1)}^{(t)} + a_3 x_{(i,j-1)}^{(t)} + a_4 x_{(i-1,j-1)}^{(t)} \\ &\quad + a_5 x_{(i-1,j)}^{(t)} + a_6 x_{(i-1,j+1)}^{(t)} + a_7 x_{(i,j+1)}^{(t)} + a_8 x_{(i+1,j+1)}^{(t)} \pmod 3, \end{aligned} \tag{2.1}$$

where $a_0, a_1, \ldots, a_8 \in \mathbb{Z}_3$. The value of each cell for the next state may not depend upon all nine neighbors. Regarding the neighborhood of the boundary cells (see [3] for details), two approaches exist:

- If the boundary cells are connected to 0-state (see Table 1), then CA are called NB CA.
- If the boundary cells are adjacent to each other (see Table 3), then CA are called PB CA.

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