Applied Mathematical Modelling 000 (2016) 1–17

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

On the parametric uncertainty quantification of the Rothermel's rate of spread model

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A R T I C L E I N F O

Article history:
Received 10 October 2013
Revised 1 June 2016
Accepted 16 June 2016
Available online xxx

Keywords:
Stochastic model
Non-Intrusive Spectral Projection
Polynomial chaos expansion
Fire spread
Rothermel's model

A B S T R A C T

Parametric uncertainty quantification of the Rothermel's fire spread model is presented using the Polynomial Chaos expansion method under a Non-Intrusive Spectral Projection (NISP) approach. Several Rothermel's model input parameters have been considered random with an associated prescribed probability density function. Two different vegetation fire scenarios are considered and NISP method results and performance are compared with four other stochastic methodologies: Sensitivity Derivative Enhance Sampling; two Monte Carlo techniques; and Global Sensitivity Analysis. The stochastic analysis includes a sensitivity analysis study to quantify the direct influence of each random parameter on the solution. The NISP approach achieved performance three orders of magnitude faster than the traditional Monte Carlo method. The NISP capability to perform uncertainty quantification associated with fast convergence makes it well suited to be applied for stochastic prediction of fire spread.

1. Introduction

Over the last 50 years several models have been proposed to predict wildland fire spread [1]. The semi-empirical Rothermel's model [2] has received by far the most acceptance and many software tools, such as Behave [3], Farsite [4] and FlamMap [5], have incorporated the model to predict the rate of spread, the reaction intensity and the flame length. The Rothermel's model requires input parameters grouped by fuel data (fuel type and fuel moisture), topography (slope and aspect), and weather conditions (wind velocity and direction). The advances of multidimensional Computational Fluid Dynamics originate another type of models based on the local modeling of the physical fluid flow, heat and mass transfer as well as local combustion processes [6–8]. The empirical models are computationally fast while the physical models are far too slow to be used for real time scenarios. However, the semi-empirical models are not used for real time mitigation of wildland forest fires.

We believe that the uncertainty issue is an important reason why modeling is still not used in fire mitigation. Although the computational time increases dramatically there are already solutions to speed up the calculations, for example, by the use of Graphical Processors Units (GPUs) [9]. So it is feasible to combine these architectures with stochastic calculations to get faster than real time stochastic fire spread simulations [10].

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http://dx.doi.org/10.1016/j.apm.2016.06.026

There are too many uncertainties intrinsic to a wildland forest fire spread simulation and it is well known that it is unrealistic to assume the input data to be certain, whereby the intrinsic uncertainty of the input parameters should be considered if a realistic approach is required. Consequently, a stochastic procedure should be carried out for fire spread uncertainty quantification.

The main source of parametric uncertainty in forest fire modeling is related with the temporal and spatial mutability of input data. For instance, fuel type and fuel moisture content have a major contribution to the fire spread rate and the smallest variability can significantly accelerate or retard the fire front. The wind conditions usually present large uncertainty levels and strongly influence the fire spread velocity and direction.

Uncertainty quantification usually means the quantification of the probability of the outcome as a function of the considered parametric uncertainties. The manner in which these uncertainties influence the final outcome of the simulation is nontrivial since nonlinearities are present. Thus, several methodologies along the process must be accurately established according to the study required.

The sensitivity analysis and the uncertainty propagation are rarely performed for wildland forest fire simulation and the literature is scarce. Albeit at high computational cost, Monte Carlo (MC) simulations [11] are very robust and can be used for verification purposes against other methods. Apart from MC, other methods have been applied to forest fire research. The perturbation method expands all the stochastic quantities around their mean via Taylor series. An analytic method using first order Taylor series [12] was developed in the context of fire risk analysis. This method is computationally cheaper than MC because the implementation work has to be done just once and can then be applied to any input data set without great effort. However, the resulting system of equations becomes complex beyond second-order expansion, limiting the Taylor series approach to small perturbations and not allowing it to provide information on high-order statistics of the response.

Focusing on parametric uncertainty for sensitivity analysis studies under the Rothermel's equations system, several methodologies arise in the literature, namely the Sensitivity Derivative Enhance Sampling (SDES) and the Global Sensitivity Analysis (GSA). The SDES [13] exploits the sensitivity derivative information to accelerate the convergence of the classical MC method, revealing it to be an efficient method in quantifying the impact of input uncertainties onto the output results. The variance-based GSA [14] method uses slight modifications of the input conditions and provides sensitivity indices for original and modified scenarios, allowing a quantitative ranking of the variables and, consequently, determining the influence of each random variable on the output solution.

An alternative approach applied to the widely used semi-empirical Rothermel's model in wildland forest fire simulation is discussed in this work, based on a spectral stochastic description of uncertain parameters and field quantities. This spectral methodology makes use of Polynomial Chaos (PC) expansion [15,16] and the required probabilistic information about the stochastic solution (e.g. statistics, Confidence Intervals (CIs), Probability Density Functions (PDFs) or sensitivity to parametric uncertainty) is extracted from the coefficients of the PC expansion [17,18].

The Non-Intrusive Spectral Projection (NISP) approach uses the deterministic code as a black box and the PC coefficients of the stochastic model solution are calculated *a posteriori*. The stochastic process is constructed from evaluations of deterministic functions at an optimal number of points defined in the input support space. So, the deterministic model is computed for different sample sets of the uncertain input parameters, which follow a post-processing method in order to quantify the uncertainty propagation through the model.

The main objective of this paper is to analyze the NISP method performance and to quantify the uncertainty of each individual input parameter into the final solution, which permits a detailed sensitivity analysis. For this purpose, two different scenarios are considered and the results are compared with other stochastic methodologies published in the open literature.

The next section describes the problem formulation followed by the stochastic methods employed in the present work, namely the MC techniques used (pseudo-random and quasi-random number generator) and the PC expansion method, focusing on the NISP approach. The results section presents the uncertainty quantification of the Rothermel's model and is divided into two distinct scenarios. For both scenarios the input conditions are established by a set of random input parameters with associated PDF. In Scenario 1 the NISP approach is compared with Sensitivity Derivative Enhance Sampling (SDES) method [13] as well as with two different MC techniques. The second scenario presents a comparison between NISP and Global Sensitivity Analysis (GSA) results [14] and discusses the influence of each uncertain input parameter into the output results through a detailed analysis of output PDF. For both scenarios, a sensitivity analysis of the random input parameters is made based on the output results of stochastic coefficients. The paper ends with a summary of the conclusions.

2. Problem formulation

In this study, the output variables of interest are the energy released per unit area for a specific period of time given by the reaction intensity I_R (Eq. (1)), the rate of spread *ROS* in a specific point of the fire front (Eq. (2)), the effective wind speed *EFW* (Eq. (3)) and the direction of maximum spread *SDR* (Eq. (4)). These Rothermel's outputs [2] are functions of, respectively:

$$I_R = f(\Gamma', w_n, h, \eta_M, \eta_S) \tag{1}$$

$$ROS = f(I_R, \delta, \rho_b, \varepsilon, Q_{ig}, \varphi_W, \varphi_S) \tag{2}$$

$$EFW = f(\beta, \beta_{opt}, sv, \varphi_C, B, C, E) \tag{3}$$

Please cite this article as: A.R. Ervilha et al., On the parametric uncertainty quantification of the Rothermel's rate of spread model, Applied Mathematical Modelling (2016), http://dx.doi.org/10.1016/j.apm.2016.06.026