Applied Mathematical Modelling 000 (2016) 1–22

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Characterization of a nonlinear MEMS-based piezoelectric resonator for wideband micro power generation

Abdolreza Pasharavesh [a], M.T. Ahmadian [a,b,*]

[a] Mechanical Engineering Department, Sharif University of Technology, Tehran, Iran
[b] Center of Excellence in Design, Robotics and Automation, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 29 December 2015
Revised 28 June 2016
Accepted 14 August 2016
Available online xxx

Keywords:
Micro power generation
Coupled electromechanical modeling
Nonlinear resonator
Broadband energy harvesting

ABSTRACT

Micro-scale piezoelectric unimorph beams with attached proof masses are the most prevalent structures in MEMS-based energy harvesters considering micro fabrication and natural frequency limitations. In doubly clamped beams a nonlinear stiffness is observed as a result of midplane stretching effect which leads to amplitude-stiffened Duffing resonance. In this study, a nonlinear model of a doubly clamped piezoelectric micro power generator, taking into account geometric nonlinearities including stretching and large curvatures, is investigated. The governing nonlinear coupled electromechanical partial differential equations of motion are determined by exploiting Hamilton's principle. A semi-analytical approach implementing the perturbation method of multiple scales is used to solve the nonlinear coupled differential equations and analyze the primary and superharmonic resonances. Results indicate that operational bandwidth of the nonlinear harvester is enhanced considerably with respect to linear models. Moreover considerable amount of power is generated due to occurrence of superharmonic resonances. This yields to extraction of energy at subharmonics of the natural frequency which is crucially important in MEMS-based harvesters.

1. Introduction

While kinetic energy harvesters as an efficient alternative for former heavy batteries of limited service lives have been of interest for consecutive years, recent progresses accomplished in two tightly interconnected fields of engineering namely Micro-Electro-Mechanical Systems (MEMS) and microelectronics technologies have proposed the idea of developing MEMS-based harvesting devices to provide required energy of intelligent systems with low-power-consuming electronics. These energy autonomous systems may be exploited in remotely operating devices such as microsensor nodes of a wireless sensor network (WSN) in civil and industrial applications [1,2] or inaccessible embedded apparatus such as implantable medical devices (IMD) [3,4]. Most of already fabricated MEMS harvesters are linear resonant devices employing piezoelectric materials in the energy conversion process [5–8]. Piezoelectric harvesters according to their high energy density storage capability [9] and less complicated fabrication processes [10] are preferred in micro scale to their electromagnetic [11,12] and electrostatic [13,14] counterparts.

The major challenge of linear resonant energy harvesters is their narrow operation frequency bandwidths which restricts the energy generation only to the excitation frequencies in an extremely narrow neighborhood of the system resonance frequency. Furthermore when a linear resonator is miniaturized, its resonance frequency grows larger and since in most

* Corresponding author at : Mechanical Engineering Department, Sharif University of Technology, Tehran, Iran. Fax: +98 21 66000021.
E-mail address: ahmadian@sharif.edu (M.T. Ahmadian).

http://dx.doi.org/10.1016/j.apm.2016.08.019
0307-904X/

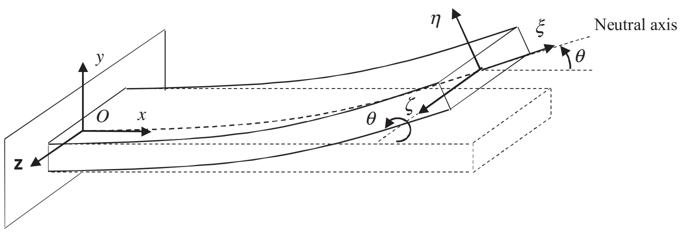

Fig. 1. A segment of the beam before and after deflection.

cases it is much higher than the frequency of environmental vibration sources, it will be poorly suited for energy harvesting in many situations. Low efficiencies of energy conversion in MEMS-based piezoelectric harvesters is another challenge of these devices arising from decline in electromechanical coupling coefficient of piezoelectric materials during conventional deposition processes. Although during the recent years researchers have partly succeeded in fabricating new MEMS devices with high generated power at low frequencies by improving the design and fabrication processes [15–17], there are rare publications on fabrication of wideband MEMS harvesters [18].

Several bandwidth broadening approaches have been purposed to circumvent this so-called "Gain-Bandwidth Dilemma" where broadband harvesting by nonlinear mechanisms is one of these strategies [19]. Deliberate introduction of nonlinearities can lead to performance improvement of harvesting devices with both single harmonic and broadband random excitations [20]. Mann and Sims [21], Quinn et al. [22] and Barton et al. [23] showed that adding nonlinearities to a resonant harvester leads to bandwidth improvement in response to single harmonic excitations through bending of the response curves of the system which extends the coupling between the response of the harvester and the excitation to a wider range of frequencies. While monostable harvesters with Duffing-type nonlinearities are shown to be ineffectual in enhancement of power output under random excitations [24], a randomly excited bistable device through the activation of noise-assisted jumps between energy wells can significantly improve the output power and have been investigated by many researchers during the recent years [25–27].

The first ultra-wide bandwidth piezoelectric nonlinear MEMS harvester was fabricated by Hajati and Kim [28]. They utilized the stretching strain in a doubly clamped beam instead of the bending strain in a cantilever structure to develop their harvester. Nonlinear stiffness due to the midplane stretching in a doubly clamped beam results in an amplitude-stiffened Duffing resonance. Their experimental findings show that more than one order of magnitude of improvement in comparison to the previously reported linear devices in both bandwidth and power density is obtained. Although inherent sources of nonlinearity including material nonlinearities [29], integration to nonlinear circuits of energy extraction [30] and geometrical nonlinearities [31] can affect the behavior of piezoelectric harvesters, previous efforts on the analysis of harvesters constructed of laminated piezoelectric beams were mostly devoted to linear problems [32–39].

The main aim of this paper is to fully characterize the nonlinear behavior of a piezoelectric vibrational harvester with the capability of being fabricated in micro scale implementing uncomplicated MEMS fabrication processes. To achieve this goal a coupled electromechanical nonlinear model of a clamped-clamped piezoelectric unimorph beam with an attached proof mass considering nonlinearities due to midplane stretching and large curvatures is presented. Governing equations are extracted through Hamilton's principle and solved using some perturbation technique. The capability of the harvester in wideband energy scavenging from both primary and superharmonic resonances is investigated and discussed in detail to show its potential to alleviate two above-mentioned challenges of MEMS harvesters, namely narrow operational bandwidths and high resonance frequencies.

2. Theory and formulation

Schematic of a beam segment is shown in Fig. 1 where xyz denotes the reference coordinate system connected to the clamped end of the beam with orthogonal unit vectors ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$), while $\xi\eta\zeta$ denotes a local curvilinear coordinate system at length s in the deformed position with orthogonal unit vectors ($\mathbf{e}_\xi$, $\mathbf{e}_\eta$, $\mathbf{e}_\zeta$). Moreover, x and ξ axes represent the neutral axis of the beam before and after the deformation, respectively. According to Euler–Bernoulli beam theory, all planes which are perpendicular to neutral axis before deformation will remain plane and perpendicular to neutral axis after the beam deflects [40].

Assuming that the motion of the beam occurs in xy plane and the beam deflects about ζ axis, denoting the rotation angle of the cross section by θ, the unit vectors of two coordinate systems are related by the following relation:

$$\begin{Bmatrix} \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z \end{Bmatrix} = [T] \begin{Bmatrix} \mathbf{e}_\xi \\ \mathbf{e}_\eta \\ \mathbf{e}_\zeta \end{Bmatrix} \quad [T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

According to Fig. 2 one obtains:

$$\sin\theta = \frac{v'}{1+e} \qquad \cos\theta = \frac{1+u'}{1+e} \qquad \tan\theta = \frac{v'}{1+u'}, \tag{2}$$

Please cite this article as: A. Pasharavesh, M.T. Ahmadian, Characterization of a nonlinear MEMS-based piezoelectric resonator for wideband micro power generation, Applied Mathematical Modelling (2016), http://dx.doi.org/10.1016/j.apm.2016.08.019