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Laplace transform–homotopy perturbation method with arbitrary initial approximation and residual error cancelation

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This paper presents a modified Laplace transform homotopy perturbation method with finite boundary conditions (MLT–HPM) designed to improve the accuracy of the approximate solutions obtained by LT–HPM and other methods. To this purpose, a suitable initial approximation will be introduced, in addition, the residual error in several points of the interest interval (RECP) will be canceled. In order to prove the efficiency of the proposed method a couple of nonlinear ordinary differential equations with mixed boundary conditions, indeed, difficult to approximate, are proposed. The square residual error (S.R.E) of the proposed solutions will result to be of hundredths and tenths, requiring only a first order approximation of MLT–HPM, unlike LT–HPM, which will require more iterations for the same cases study.

1. Introduction

Besides its theoretical interest, the Laplace transform (L.T.) has played an important role in mathematics, because its application allows solving, in a simple fashion, many problems in science and engineering [1]. As it is well known the Laplace transform is useful for solving linear ordinary differential equations with constant coefficients and initial conditions as well as some cases of differential equations with variable coefficients and partial differential equations [1]. Moreover, the applications of L.T. for nonlinear ordinary and partial differential equations mainly focus to find approximate solutions, thus in [2] was reported a combination of homotopy perturbation method (HPM) and L.T. methods (LT–HPM), in order to solve approximately, nonlinear problems with initial conditions [2,3]. On the one hand, LT–HPM was adopted with the purpose to apply it to the case of nonlinear problems with boundary conditions defined on finite intervals [4–7]. In particular Filobello-Nino et al. [6] introduced a modified version of LT–HPM, the nonlinearities distribution Laplace transform–homotopy perturbation method (NDLT–HPM) and Filobello-Nino et al. [7] proposed an extension of Laplace transform–homotopy perturbation method to solve nonlinear differential equations with variable coefficients. On the other hand, the case of equations

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with boundary conditions on infinite intervals has been studied in some articles and correspond often to problems defined on semi-infinite ranges [8] however, the methods for solving those problems are different from the methods presented in this paper. In the same way [9] employed homotopy perturbation method (HPM) coupled with Laplace transformation and He's polynomials in order to include the case of linear and nonlinear partial differential equations (PDES). At present [10], the coupling of HPM with Laplace transform has been widely used to solve fractional differential equations, as it has been reported by the Refs. [11–19].

Following the previous review, this work proposes the modified Laplace transform homotopy perturbation method (MLT–HPM) which may be considered a continuation of Filobello-Nino et al. [4]. Thus the content of this paper refers to the case of problems with boundary conditions defined on finite intervals. As it will be seen, the proposed method follows a strategy which on the one hand introduces an arbitrary initial trial function and on the other hand proposes to cancel the residual error in several points of the interval of interest. It will be seen that the above procedure will accelerate the convergence of the proposed solution. Later on, two cases of nonlinear problems with finite mixed boundary conditions will be introduced which are indeed difficult to model not only for LT–HPM, but for other methods, as it will be shown in this work.

The proposed results will show the potential of MLT–HPM in the search for analytical approximate solutions for ODES under the mentioned conditions.

As it was above mentioned [9] employed homotopy perturbation method (HPM) coupled with the Laplace transformation and He's polynomials in order to solve linear and nonlinear PDES without specifying special requirements about boundary conditions. Unlike the previously mentioned methods, MLT–HPM is a modification of LT–HPM implemented specifically to solve ODES with finite boundary conditions. For this purpose, MLT–HPM introduces a suitable initial approximation and cancels the residual error in several points of the interest interval in order to improve its efficiency. From the above, it is clear that He–Laplace method and MLT–HPM are quite different, even though they employ Laplace transformation. Besides, MLT–HPM methodology does not require the use of He's polynomials.

The importance of research on nonlinear differential equations is that many phenomena, practical or theoretical, are of nonlinear nature. For this reason, several methods focused to find approximate solutions for nonlinear differential equations have been reported such as those based on variational approaches [20], tanh method [21], exp-function [22], Adomian's decomposition method (ADM) [23], parameter expansion [24], homotopy perturbation method (HPM) [2–4, 8,25–28,5,6,10], homotopy analysis method (HAM) [29–31], parameterized perturbation method (PPM) [32], lattice Boltzmann method (LBM) [33], reconstruction of variational iteration method (RVIM) [34,35] and perturbation method [36–38] among many others.

Particularly, PPM [32] is an effective and powerful method employed to find analytical approximate solutions for ODES.

The method introduces and artificial parameter and it assumes that the solution for a problem is expressed as a power series solution through the mentioned parameter. Unlike MLT–HPM, as it will be seen, introduces and optimally calculates some parameters in order to get accurate solutions; PPM method provides approximate solutions, but eliminating completely the aforementioned parameter of the final solution.

Also, ADM is a method employed in order to find approximate solutions for nonlinear problems in science and engineering [23]. An advantage of this method is that, it provides analytical approximate solutions without any simplifications like linearization, perturbation parameters, closure approximations, among others. Nevertheless, unlike MLT–HPM method which we will see, employs basic properties of Laplace transform in order to obtain analytical approximate solutions to ODES; ADM requires of the calculation of the thus called Adomian polynomials. Nevertheless this is a procedure generally long and cumbersome.

One of the useful numerical methods used recently is the LBM [33]. This method has been used in order to study some phenomena related with natural convection in presence of magnetic field such as the simulation the flow field in wide ranges of engineering applications [33]. However, unlike MLT–HPM which provides analytical and quantitative approximate solutions (see Section 3) the LBM only provides a qualitative description of the phenomena under study.

On the other hand RVIM, is an effective method employed in dealing with nonlinear equations [34,35]. Essentially RVIM is an alternative method for finding the optimal value of the Lagrange multiplier for VIM method, by using of Laplace transform [34]. In particular, RVIM has turned out a powerful tool to find explicit analytical solutions for many problems in science and engineering, which involve the solution of partial differential equations. We will see in Section 3 that MLT–HPM is a method which combines HPM and Laplace transform to obtain analytical approximate solutions for ODES. The case of partial differential equations requires a review of the proposed method MLT–HPM.

The paper is organized as follows. In Section 2, the basic idea of standard HPM method is introduced. For Section 3 a modified Laplace transform homotopy perturbation method is presented. Additionally, Section 4 presents two cases study. Besides, a discussion on the results is presented in Section 5, A brief conclusion is given in Section 6 and finally Appendix A is introduced in order to improve the physical insight of the first case study.

2. Standard HPM

The standard homotopy perturbation method (HPM) was proposed by He; it was introduced as a powerful tool to approach various kinds of nonlinear problems. The homotopy perturbation method (HPM) is considered as a combination of the classical perturbation technique and the homotopy (whose origin is in the topology), but it is not restricted to small parameters as it occurs with traditional perturbation methods. For example, HPM method requires neither small parameter nor linearization, but only few iterations to obtain highly accurate solutions [26].