# Coefficient identification in the Euler-Bernoulli equation using regularization methods 

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#### Abstract

In this paper, we will study the inverse problem of identification of flexural rigidity coefficient in the Euler-Bernoulli equation. This inverse problem is ill-posed. To solve it, we will use regularization methods. In particular, we will apply the mollification method and the Landweber iteration method, in particular, to find the regularized solution of the MoorePenrose generalized inverse to a linear operator and with this, we reconstruct the coefficient. At the end of this paper, will present some examples of interest.


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## 1. Introduction

When we study beam theory in strength of materials, we are interested in solving the following problem: if the transversely distributed load $f(x)$ and the flexural rigidity $a(x)$ are known in the interval [ 0,1 ], find the deflection of the beam $u(x)$ with respect to a neutral axis, when the beam satisfies the following conditions:

1. It satisfies Hooke's law; i.e., the material is linear and elastic.
2. The plane sections remain plane and perpendicular to the neutral axis.
3. Deflections are small compared with its length.
4. $u(x) \in C^{4}[0,1]$.

If $f(x) \in C[0,1]$ and $a(x) \in C^{2}[0,1]$ then $u(x)$ satisfies the Euler-Bernoulli equation:

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}}\left(a(x) \frac{d^{2} u}{d x^{2}}\right)=f(x), \quad 0 \leq x \leq 1 \tag{1}
\end{equation*}
$$

under the conditions:

$$
\begin{align*}
& \gamma_{0}=u(0),  \tag{2}\\
& \gamma_{1}=u(1), \tag{3}
\end{align*}
$$

[^0]\[

$$
\begin{align*}
& \gamma_{2}=u^{\prime \prime}(0) a(0),  \tag{4}\\
& \gamma_{3}=\left(a u^{\prime \prime}\right)^{\prime}(0) . \tag{5}
\end{align*}
$$
\]

We can deduce that the deflexion of the beam (see [1]) is given by:

$$
\begin{equation*}
u(x)=\int_{0}^{1} G(x, t) \frac{\int_{0}^{t} \int_{0}^{s} f(w) d w d s+\gamma_{3} t+\gamma_{2}}{a(t)} d t+\left(\gamma_{1}-\gamma_{0}\right) x+\gamma_{0} \tag{6}
\end{equation*}
$$

where:

$$
G(x, t)=\left\{\begin{array}{lll}
x(t-1) & \text { if } & x \leq t  \tag{7}\\
t(x-1) & \text { if } & x \geq t
\end{array}\right.
$$

In this paper, we are interested in this inverse problem: given $f(x) \in C[0,1]$ and $u(x) \in C^{4}[0,1]$, find $a(x)$. Note that, if we suppose that $\inf _{x \in I}\left|u^{\prime \prime}(x)\right|>0$ and the conditions (4) and (5) are known, we obtain that:

$$
\begin{equation*}
a(x)=\frac{\int_{0}^{x}\left(\int_{0}^{t} f(s) d s+\gamma_{3}\right) d t+\gamma_{2}}{\frac{d^{2} u}{d x^{2}}} \tag{8}
\end{equation*}
$$

which means that the problem is solved. However, in practice, we only know the function $u$ and $f$ in a discrete set of points $x=x_{i}, i=0,1, \ldots, n$ and these data (denoted as $u^{\delta}$ and $f^{\delta}$ ) have noise; that is,

$$
\begin{align*}
& u^{\delta}\left(x_{i}\right)=u\left(x_{1}\right)+\delta \theta_{i}, i=0,1, \ldots, n  \tag{9}\\
& f^{\delta}\left(x_{i}\right)=f\left(x_{i}\right)+\delta v_{i}, i=0,1, \ldots, n \tag{10}
\end{align*}
$$

Where $\theta_{i}$ and $\delta_{i}$ have values in $[0,1]$. The difficulty is that we cannot use (8) when we have noisy data because the numerical differentiation is an unstable problem. In this paper, we will discuss this problem as follows: in Section 2, we show the inherent problems in the identification problem. Then, in Section 3, we present the two regularization methods that we will use, and after that, we will use those methods to stabilize the second derivative of the function $u$ in Section 4. In Section 5, we will discuss the error estimate for the coefficient using the two methods, and finally, in Section 6 we will present some examples to show how the method is working.

## 2. Ill-posed identification problem in Euler-Bernoulli equation

When we have noisy data, we cannot use (8) to calculate $a(x)$ for the following reasons:

1. To reconstruct $a(x)$ using (8), we have the problem of finding $\frac{d^{2} u}{d x^{2}}$ in the interval $[0,1]$, we usually use central difference method to calculate that derivative, but it is known that the numerical differentiation is an ill-posed inverse problem.
2. We can have the situation where in a finite set of points $x_{i} \in(0,1), \frac{d^{2} u}{d x^{2}}\left(x_{i}\right)=0$. Then, if $\frac{d^{3} u}{d x^{3}}\left(x_{i}\right) \neq 0$ :

$$
a\left(x_{i}\right)=\frac{\int_{0}^{x_{i}} f(t) d t+\gamma_{1}}{\frac{d^{3} u}{d x^{3}}\left(x_{i}\right)} .
$$

Also, it could have $\frac{d^{2} u}{d x^{2}}\left(x_{i}\right)=\frac{d^{3} u}{d x^{3}}\left(x_{i}\right)=0$ and $\frac{d^{4} u}{d x^{4}}\left(x_{i}\right) \neq 0$. In this case, we can calculate $a\left(x_{i}\right)$ by:

$$
a\left(x_{i}\right)=\frac{f\left(x_{i}\right)}{\frac{d^{4} u}{d x^{4}}\left(x_{i}\right)} .
$$

3. Finally, if $\frac{d^{2} u}{d x^{2}}\left(x_{i}\right)=\frac{d^{3} u}{d x^{3}}\left(x_{i}\right)=\frac{d^{4} u}{d x^{4}}\left(x_{i}\right)=0$ and $\frac{d^{5} u}{d x^{5}}\left(x_{i}\right) \neq 0$, we need a differentiable $f$. If $f$ satisfies that condition, we can reconstruct $a\left(x_{i}\right)$ as:

$$
a\left(x_{i}\right)=\frac{\frac{d f}{d x}\left(x_{i}\right)}{\frac{d^{5} u}{d x^{5}}\left(x_{i}\right)} .
$$

## 3. Regularization method: basic principles

In this section, we will present two methods that we will apply to stabilize the second derivative of $f$. When we have noisy discrete data, we suppose that:

$$
\begin{gather*}
\left\|u-u^{\delta}\right\|_{\infty} \leq \delta,  \tag{11}\\
\left\|f-f^{\delta}\right\|_{\infty} \leq \delta . \tag{12}
\end{gather*}
$$

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