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Numerical optimization applying trust-region reflective least squares algorithm with constraints to optimize the non-linear creep parameters of soft soil

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ABSTRACT

Determination of the creep model parameters is a challenging task particularly when a non-linear elastic visco-plastic (EVP) model is adopted, mainly due to the limited test duration as well as the assumption of the reference time. Therefore, this paper presents an innovative numerical solution to find the EVP model parameters applying the trust-region reflective least square optimization algorithm. The developed approach involves several available laboratory consolidation test results in the optimization procedure with the adopted commencing time to creep as a unit of time. In this paper, the laboratory results of Ottawa clay were employed to demonstrate the limitation of the recent method to obtain model parameters. Furthermore, the developed method is verified against Skå-Edeby clay in the laboratory conditions. The EVP model parameters are obtained by applying the developed method to the available laboratory consolidation results of clay samples. The analysis results of vertical strains and excess pore water pressures demonstrate that the developed method can be a feasible tool to estimate the settlement properties of clays.

1. Introduction

Soft soils are generally categorized as problematic soils due to their excessive long-term settlement under surcharge load, which has been observed both in laboratory and in-situ conditions. Creep compression (i.e. time-dependent compression under a constant effective stress) due to the viscous property of soft soils is an important issue and cannot be neglected in design and construction stages.

Estimation of creep compression has been simplified by adopting a constant value $C_{\alpha e} = -\Delta e/\Delta log(t)$ or $C_{\alpha\varepsilon} = \Delta\varepsilon/\Delta log(t)$ with e as the void ratio, ε as the strain and t as the creeping time to express the relationship between creep compression with time [1–3]. It should be noted that the convention of compression positive for stresses and strains is adopted in this paper. Applying a constant $C\alpha$, creep compression continuously increases with log-time, which can unrealistically lead to infinitive settlement or a negative value of void ratio as time increases to infinity. Moreover, the laboratory measurements as well as field observations have indicated that the creep rate decreases non-linearly with log-time (i.e. C_α is not constant).

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[4–7]. The time-dependent behavior of soft soils can significantly influence the performance of structures not only under the normal conditions, but also under extreme loads such as earthquake [8–11].

Yin [7] updated a linear creep function to a non-linear creep function [i.e. Eq. (1)] to capture the decreasing creep rate with ln-time in order to overcome the limitation of the linear logarithm creep function. In Yin's [7] model, the non-linear creep function involves stress-dependent model parameters (i.e. the creep strain limit ε^{vp}_{lm} and the creep coefficient ψ_o in V-ln$t$ space in which V is the specific volume), which decrease against the increase of effective stress. The creep coefficient ψ_o/V can be correlated to the initial creep strain rate under the vertical effective stress σ'_z. The creep parameter ψ in Eq. (1) is to describe the change of creep strain with ln-time in V-ln$t$ space. Based on Eq. (1), ψ/V, which is correlated to $C_{\alpha\varepsilon}/\ln 10$ (i.e. $\Delta\varepsilon_{creep} = C_{\alpha\varepsilon} \times \Delta \log t \approx \psi/V \times \Delta \ln t \Rightarrow \psi/V \approx C_{\alpha\varepsilon}/\ln 10$), may decrease not only with effective stress but also with time. Since the creep parameter ψ/V, and the traditional secondary compression index $C_{\alpha\varepsilon}$ are obtained through two different methods with different suggested procedures for parameter determination, their values are approximately correlated in the same loading conditions. The equivalent time t_e is the elapsed creep time starting from the reference time t_o to the current stress–strain point [12].

$$\varepsilon^{vp}_z = \frac{\psi}{V} \ln\left(\frac{t_o + t_e}{t_o}\right) = \frac{\psi_o{}^*}{1 + \frac{\psi_o{}^*}{\varepsilon^{vp}_{lm}} \ln\left(\frac{t_o+t_e}{t_o}\right)} \ln\left(\frac{t_o + t_e}{t_o}\right), \tag{1}$$

where, is the vertical creep strain, $\psi_o{}^* = \psi_o/V$, in which $V = 1 + e$ is the specific volume and e is the void ratio.

The difficulty of this EVP model is related to the parameter determination. In order to define the stress dependency of and $\psi_o{}^*$, adopting the computational procedure developed by Yin [7], at least 1-week test duration for each stage is required. Besides, the reference time value of t_o should be chosen in advance. Although Yin [7] mentioned any value of t_o can be selected in advance, no computational procedure was clearly proposed. It can be difficult to define the accurate value of the effective stress, when t_o is less than the time at the end of primary consolidation. Thus, the relationships between the creep parameters and the effective stress cannot be readily established. For simplification, Yin [7] recommended to adopt $t_o = t_{EOP}$ (i.e. the time at the end of primary consolidation). Since $t_o = t_{EOP}$, the value of t_o can vary with the soil sample thickness or the drainage condition [13]. Moreover, various stress levels may result in different values of t_{EOP}. The value of t_o indicates the starting point of creep compression. Thus, $t_o = t_{EOP}$ implies that creep compression starts after the end of primary consolidation, which is against the definition of the reference time line which is a viscous free line and the supposition that creep starts from the beginning of loading. Besides, based on the parameter curve fitting explained in Yin [7], t_o may also influence the values of creep parameters, since t_o is used as an input value to determine other parameters.

The time-dependent behavior of soils occurs in saturated and unsaturated soils [14,15], and is influenced by various factors such as structural viscosity, soil cementation and its degradation [16,12]. Conventionally, numerical optimization methods have rarely been utilized to determine the model parameters in geotechnical engineering. Especially for soft soil creep, model parameters have been determined using simple graphical procedure with many assumptions. For example, as mentioned above, the reference time t_o should be adopted in advance as recommended by Yin [7], it can be assumed as the time at the end of primary consolidation, which is observed not to be a unique value. In this paper, the method suggested by Yin was applied to obtain the model parameters of Ottawa Clays by using several choices of t_o in order to show the importance of choosing t_o. Meanwhile, numerical optimization methods have been widely used in other disciplines such as structural engineering, transport, and material [17–20]. Therefore, this paper presents a numerical approach to estimate several EVP model parameters to enhance the merit of the non-linear creep function as well as overcome the difficulty of parameter determination. The numerical approach applies the advanced trust-region reflective least square (TRRLS) algorithm to optimize a set of EVP parameters by involving several loading stages of consolidation test results. The trust-region method is one of the best optimization methods for nonlinear problems [21,22]. Along with the interior-reflective algorithm, the nonlinear least squares problem can be solved with bound constraints [23]. In this study, applying the TRRLS approach for the model parameter optimization, all available test data during and after the end of primary consolidation can be utilized in the optimization process, since t_o in this approach is adopted as a unit of time. Thus, t_o is independent of the soil sample thickness, drainage condition and stress level. The unit of time is chosen based on the time unit of the data. In this paper, t_o is adopted as 1 min. The developed method has been efficiently applied in the laboratory condition [24,25]. Moreover, in order to evaluate the developed approach, the available laboratory test results of Skå-Edeby clays are used to obtain the EVP model parameters using the developed method for different drainage conditions.

2. Elastic visco-plastic model with non-linear creep function

The elastic visco-plastic model proposed by Yin & Graham [26] includes a series of time lines as shown in Fig. 1. An instant time-line describes the time-independent elastic behavior (Eq. 2), and a reference time-line represents the time-independent elastic-plastic behavior (Eq. 3). The elastic visco-plastic model proposed by Yin & Graham [26] can be applied for one-dimensional (1D) and three-dimensional (3D) conditions. For 3D consolidation conditions, the model parameters including κ^*, λ^*, and $\psi_o{}^*$ are obtained based on the volumetric strains under isotropic loading [27]. On the other hand, for 1D condition, the model parameters are defined based on vertical strains. In this study, 1D consolidation theory is applied, and the model parameters are defined based on vertical strains. Since in this case study, only oedometer test results have been available, authors have used 1D consolidation theory. The subscript z is used to indicate the vertical direction. Thus