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A hybrid self-adjusted mean value method for reliability-based design optimization using sufficient descent condition

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ABSTRACT

Due to the efficiency and simplicity, advanced mean value (AMV) method is widely used to evaluate the probabilistic constraints in reliability-based design optimization (RBDO) problems. However, it may produce unstable results as periodic and chaos solutions for highly nonlinear performance functions. In this paper, the AMV is modified based on a self-adaptive step size, named as the self-adjusted mean value (SMV) method, where the step size for reliability analysis is adjusted based on a power function dynamically. Then, a hybrid self-adjusted mean value (HSMV) method is developed to enhance the robustness and efficiency of iterative scheme in the reliability loop, where the AMV is combined with the SMV on the basis of sufficient descent condition. Finally, the proposed methods (i.e. SMV and HSMV) are compared with other existing performance measure approaches through several nonlinear mathematical/structural examples. Results show that the SMV and HSMV are more efficient with enhanced robustness for both convex and concave performance functions.

1. Introduction

In practice, there exist a lot of uncertain factors (e.g. dimensions, models, materials and loads) in real-world engineering. The reliability-based design optimization (RBDO) can provide an optimal design to achieve a good balance between total cost and confidence level under uncertainties. In general, there are three approaches to solve RBDO problems, including double loop approach (DLA), single loop approach (SLA), and decoupled loop approach. For the DLA, the inner loop provides reliability information for deterministic optimization based on the outer loop in each cycle [1,2]. The DLA can be regarded as an accurate procedure to determine the optimum design, but it is computationally inefficient because the evaluation of performance function is required in each inner loop. Therefore, a more efficient reliability method is crucial to evaluate the reliability level of probabilistic constraints in RBDO problems [3].

For the DLA, reliability information can be obtained in two different ways, including reliability index approach (RIA) [4,5] and performance measure approach (PMA) [6]. It has been generally recognized that the PMA has higher efficiency and robustness in comparison with the RIA [7,8]. For the PMA, the probabilistic constraint is evaluated by an iterative FORM

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formula, e.g. the advanced mean value (AMV), conjugate mean value (CMV) [1], hybrid mean value (HMV) [9], enhanced hybrid mean value (EHMV) [2], chaos control (CC) [8], modified chaos control (MCC) [10], hybrid modified chaos control (HCC) [10], adaptive chaos control (ACC) [11], conjugate gradient analysis (CGA) [12], Relaxed mean value [13] and self-adaptive modified chaos control (SMCC) [14] methods. In general, the AMV has been widely used due to its simplicity and efficiency, however, it may produce periodic, bifurcate or chaotic solutions for highly nonlinear performance functions [11,14]. In this case, the HMV and CMV usually do not converge [10,12], and the iterative formula of CMV, HMV and EHMV are computationally inefficient. In addition, the CC requires a large number of iterations to achieve stabilization for either concave or convex performance functions. The MCC and CGA are inefficient for convex performance functions. The SMCC was established based on a dynamic chaos control between zero and maximum step size less than 1, thus the SMCC may be less efficient than the AMV for convex performance functions. The dynamic step size in the SMCC depends on the reliability index, performance function at the new and previous points, therefore, it may cause a rigorous dynamic chaos control for complex engineering problems. Consequently, the SMCC may be slowly converged for nonlinear reliability problems. Therefore, it is very crucial to develop a robust reliability analysis algorithm with reduced computational demand based on the iterative first-order reliability method, which can be further enhanced by a simple iterative scheme.

In this paper, a self-adaptive step size is proposed to improve the robustness of AMV, which is computed based on the power function. Then, the reliability analysis method to evaluate probabilistic constraints is modified on the basis of a simple iterative formula, which is called self-adaptive mean value (SMV) method. Furthermore, a hybrid self-adaptive mean value (HSMV) method is developed to enhance the efficiency of searching the minimum performance target point (MPTP), which is the combination of the proposed SMV and AMV based on sufficient descent condition. This study is organized as follows: in Section 2, several MPTP search methods (i.e. AMV, HMV, MCC and HCC) are introduced briefly. Then, two proposed reliability methods (i.e. SMV and HSMV) are developed for MPTP search, and the framework of PMA-based HSMV is illustrated in Section 3. In Section 4, three nonlinear mathematical examples and two structural examples with concave or convex performance functions are utilized to demonstrate the efficiency and robustness of the proposed methods. After that, the proposed methods are compared with several existing reliability methods through three nonlinear RBDO problems. Finally, the conclusions are drawn in Section 5.

2. Performance measure approach (PMA)

For a RBDO problem, the formulation can be expressed as [15,16]:

$$\begin{aligned}
&\text{find } \mathbf{d}, \boldsymbol{\mu}_{\mathbf{x}} \\
&\min \ f(\mathbf{d}, \mathbf{X}) \\
&s.t. \ P_f[g_j(\mathbf{d}, \mathbf{X}) \le 0] \le \boldsymbol{\Phi}(-\beta_t^j) \quad j = 1, 2, \ldots, p \\
&\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \quad \boldsymbol{\mu}_{\mathbf{x}}^L \le \boldsymbol{\mu}_{\mathbf{x}} \le \boldsymbol{\mu}_{\mathbf{x}}^U,
\end{aligned} \tag{1}$$

where f is the objective or cost function, g_j is the j_{th} performance function, β_t^j is the target reliability index for the j_{th} probabilistic constraint, p is the number of performance functions, $\boldsymbol{\Phi}$ is the standard normal cumulative distribution function. Two types of variables include deterministic design variables $\mathbf{d}$ (representing physical quantities with lower bound $\mathbf{d}^L$ and upper bound $\mathbf{d}^U$) and random variables $\mathbf{X}$ (representing uncertain quantities with lower bound $\boldsymbol{\mu}_{\mathbf{x}}^L$ and upper bound $\boldsymbol{\mu}_{\mathbf{x}}^U$). The failure probability P_f in Eq. (1) can be computed by the following multi-dimensional integration [17,18]:

$$P_f[g(\mathbf{d}, \mathbf{X}) \le 0] = \int\limits_{g(\mathbf{d},\mathbf{X}) \le 0} \ldots \int f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X}, \tag{2}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function of the basic random variables $\mathbf{X}$, and $g(\mathbf{d}, \mathbf{X}) \le 0$ denotes the failure domain. By use of the cumulative distribution function (F_{g_j}) of the performance function (g_j), the above relation can be rewritten as follows [19]:

$$P\left[g_j(\mathbf{d}, \mathbf{X}) \le 0\right] = F_{g_j}(\mathbf{d}, 0) = \int\limits_{g_j(\mathbf{d},\mathbf{X}) \le 0} \ldots \int f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \approx \boldsymbol{\Phi}(-\beta_t^j), \tag{3}$$

$$g_j(\mathbf{d}, \mathbf{X}) = F_{g_j}^{-1}(\mathbf{d}, \Phi(-\beta_t^j)) \ge 0. \tag{4}$$

Due to the simplicity and efficiency, Eq. (4) is employed to evaluate the probabilistic constraints in Eq. (1) using the performance measure function, which is referred to as PMA. Thus, Eq. (1) can be rewritten by the following RBDO model for the PMA:

$$\begin{aligned}
&find \ \mathbf{d}, \boldsymbol{\mu}_{\mathbf{x}} \quad \min \ f(\mathbf{d}, \mathbf{X}) \\
&S.t. \ g_j(\mathbf{d}, \mathbf{X}) \ge 0 \quad j = 1, 2, \ldots, p \\
&\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \ \boldsymbol{\mu}_{\mathbf{x}}^L \le \boldsymbol{\mu}_{\mathbf{x}} \le \boldsymbol{\mu}_{\mathbf{x}}^U.
\end{aligned} \tag{5}$$

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