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Risk process approximation with mixing

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ABSTRACT

The approximations of risk processes with mixed exponentially distributed inter-arrival times are investigated. The number of claims in a fixed time interval is Mixed Poisson distributed. The approximating process is always overdispersed. This allows a better fit to more realistic situations in finances, than e.g. classical Cramér–Lundberg model.

The claim sizes are divided in three different groups, dependently on finiteness of their first two moments. We illustrate all the cases by numerical examples.

The case of diffusion approximation is investigated. Both American and European Pareto claims sizes are also studied.

1. Introduction

Ruin minimization has become a classical criterion in risk optimization and portfolio management, studied especially in the Basel II–VI frameworks. Several results with respect to diffusion approximation, with various conditions on first and second moments, have been received by Iglehart et al. [1–5]. In 1991, Grandel [6] generalized the idea of diffusion approximation in case when the counting process is Cox process. He supposed, however, that claim sizes have finite mean. Pancheva and Mitov [7] made survey of all mentioned results with generalizations for a non-identically distributed claim.

In this paper, we consider random time transformed Cramér–Lundberg model with arbitrary positive mixing variable and obtain corresponding approximations of the risk process. The inter arrival times are dependent. The form of dependence can be described by Archimedean survival copulas as discussed in Albrecher et al. [8]. The risk processes have been also studied with respect to non-constant interest rate (see e.g. [9–12]). Convergence rate in the case of diffusion approximation together with the distribution of underlying stochastic processes is investigated. Particular case on Poisson process with Pareto mixing variable is investigated in Jordanova and Stehlík [13]. In this setup, it is possible to obtain some explicit formula for the ruin probability by conditioning arguments.

The organization of paper is as follows. In the next section we address the probabilistic properties of Mixed Poisson process and obtain some preliminary but useful results. In Section 3, risk process and its approximation are studied. Section on limit approximation and convergence rates follows. Numerical illustrations are provided in order to better explain the obtained results.

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2. Description of the model and preliminary results

Suppose Λ is a random mixing variable with $P(\Lambda > 0) = 1$ and distribution function (d.f.) F_Λ. We consider the following novel insurance model, which special case for I type Pareto mixing variable was studied in Jordanova and Stehlík [13].

(A1) $Y_0 = 0$ and for all $n \in N$ the inter-arrival times $Y_1, Y_2, \dots Y_n$ are random vectors with mixed exponential distribution with mixing variable Λ, i.e. for all $n \in N$ their survival function is

$$P(Y_1 > x_1, \dots, Y_n > x_n) = \int_0^\infty \exp(-(x_1 + \cdots + x_n)\lambda) dF_\Lambda(\lambda).$$

$$x_1 > 0,\ x_2 > 0, \dots,\ x_n > 0. \tag{1}$$

The last implies that $EY_i = E(1/\Lambda), Var\, Y_i = E(1/\Lambda)^2$ if they exist. Laplace transformation has the form

$$Ee^{-zY_i} = E\frac{\Lambda}{\Lambda + z}, \quad z > 0, \quad i = 1, 2, \dots \tag{2}$$

(A2) Denote by $T_n = Y_0 + \cdots + Y_n$, $n = 0, 1, \dots$ the claim arrival times.
Then T_n is mixed Erlang distributed with mixing variable Λ and $n \in N$,

$$ET_n = nE\frac{1}{\Lambda}, \tag{3}$$

$$Var\ T_n = nE\frac{1}{\Lambda^2}, \tag{4}$$

$$Ee^{-zT_n} = E\left(\frac{\Lambda}{\Lambda + z}\right)^n. \tag{5}$$

which exists for all $z > 0$.

(A3) Denote by $X_0 = 0$ and $X_1, X_2, \dots$ the corresponding claim sizes. We suppose that they are i.i.d., with d.f. F, and possibly infinite variance σ^2. Assume also that they are independent on the claim arrival times.

(A4) Define the counting process

$$N^*(t) = max\{k \geq 0 : T_k \leq t\} = \sum_{k=1}^{\infty} I_{T_k}\{(0, t]\},$$

where

$$I_{T_k}\{(0, t]\} = \begin{cases} 0, & T_k \notin (0, t] \\ 1, & T_k \in (0, t] \end{cases}$$

denotes indicator random variable. $N^*(t)$ counts the number of claims in the interval (0, t].

It is a homogeneous in time Mixed Poisson process with mixing variable Λ. The last means that for any fixed $t > 0$, $N^*(t)$ is Mixed Poisson distributed r.v. with mixing variable Λ. (Briefly $N^*(t) \sim MPP(t\Lambda)$). It is well known that N^* is not a renewal process. It has the following representation:

$$N^*(t) = N(\Lambda t), \quad t \geq 0,$$

when N is a homogeneous Poisson process with intensity 1, independent on Λ.
It is easy to obtain that the counting process N^* is overdispersed, more precisely

$$\frac{VarN^*(t)}{EN^*(t)} = 1 + t\frac{Var\Lambda}{E\Lambda} \geq 1.$$

Thus our model is more realistic for difficult setups in finances than Cramér–Lundberg, which in case of finite exponential mixtures relates to upper contamination (see [14–16]). As we have observed in Potocký and Stehlík [15], Cramér–Lundberg works better only for lower contamination.

Moreover, we have $\frac{EN^*(t)}{t} = E\Lambda$.

$$Var\frac{N^*(t)}{t} = \frac{E\Lambda}{t} + Var\Lambda. \tag{6}$$

$$\frac{N^*(t)}{t} \to \Lambda,\ a.s.\ t \to \infty, \tag{7}$$

$$Ee^{-zN^*(t)} = E(e^{-\Lambda(1-\exp(-z))},$$