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Iterative solver approach for turbine interactions: application to wind or marine current turbine farms

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ABSTRACT

This paper presents a numerical investigation for the computation of wind or marine current turbines in a farm. A 3D unsteady Lagrangian vortex method is used together with a panel method in order to take into account for the turbines. In order to enforce the boundary condition onto the panel elements, a linear matrix system is defined. Solving general linear matrix systems is a topic with important scientific literature. But the main concern here is the application to a dedicated matrix which is non-sparse, non-symmetric, neither diagonally dominant nor positive-definite. Several iterative approaches were tested and compared. But after some numerical tests, a Bi-CGSTAB method was finally chosen. The main advantage of the presented method is the use of a specific preconditioner well suited for the desired application. The chosen implementation proved to be very efficient with only 3 iterations of our preconditioned Bi-CGSTAB algorithm whatever the turbine geometrical configuration. Although developed for wind or marine turbines, the proposed algorithm is absolutely not restricted to these cases, and can be applied to many others. At the end of the paper, some applications (specifically, wake computations) in a farm are presented, along with a quantitative assessment of the computational time savings brought by the iterative approach.

1. Introduction

Turbines, whether they are wind or water marine current turbines, represent a growing interest in the scientific community for energy and environmental engineering. From an historical point of view, the first studies were performed on wind turbines and recently, a similar research procedure is being developed regarding marine hydro-kinetic or marine current turbines. Furthermore, computational fluid dynamics (CFD) has been, since the beginning of these studies, a major concern for

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first aerodynamics and secondly hydrodynamics. More and more physical phenomena are being taken into account leading to an increase in the complexity of the developed numerical methods. An extremely detailed review of numerical methods dedicated to wind turbine aerodynamics and aeroelasticity was carried out by Hansen et al. [1], and a more recent one by Miller et al. [2]. A similar review for marine turbine applications was also published recently [3].

Several computational techniques exist with increasing complexity, from a classical blade element momentum (BEM) theory to a fully 3D unsteady Navier–Stokes formulation, including boundary layer treatment around the blades. Unfortunately, for the interaction of several turbines within a farm, this last approach is not really affordable at the present time; although some researchers took up the challenge [4] with impressive results. The present paper aims at describing a numerical implementation for the computation of marine current turbine hydrodynamics [5,6]. To some extent, the developed software is similar to those developed by Baltazar et al. [7] or McCombes et al. [8], also for marine current turbine applications. However, there is no real restriction to marine current turbine hydrodynamics and it may largely be used in wind energy applications [9–13]. Here, both the performances (power and thrust coefficients) and the wake are considered in an unsteady Lagrangian vortex method [14]. The blades are taken into account with a panel method using a Kutta condition for the emission of vortex particles [15]. The major advantage of such methods (that is to say panel method with free vortex blobs) is that it does not require any 3D meshing of the fluid domain, the only mesh being the one used for the discretisation of the blades. Therefore, there is no special treatment if one wants to compute several turbines in interaction, whereas classical Eulerian methods would require sophisticated meshes, probably with several rotating parts if several turbines are considered [16]. However, integral panel methods impose the resolution of a linear matrix system at each unsteady time step. The present paper aims at describing an enhanced formulation for rapid solving of such matrix systems in the case of several turbines in a farm.

Generally speaking, in Lagrangian vortex methods, should one want to treat boundary conditions, a system of linear equations appears. The present implementation [5,6] does not take dynamic stall into account, but improvements like those suggested by Voutsinas and Riziotis [10,11] make it possible to do so. In this formulation, the resolution of a matrix linear system is also required. For instance, no-slip boundary conditions can be considered with Lagrangian vortex method. This approach, developed in 2D by Ploumhans et al. [17], followed by its 3D version [18], also uses a matrix system issuing from the surface discretisation. Boundary conditions may also be enforced through Immersed Boundary (IB) and a version dedicated to velocity-vorticity formulation was proposed by Poncet [19]. To the authors' knowledge, such an implementation has never been applied to (marine or wind) turbine computations, mainly owing to their computational cost at such Reynolds numbers. However, Poncet's Immersed Boundary method [19] was applied to model the flow around a plane, which tends to give confidence in such possibilities. In some sense, the proposed numerical treatment for the resolution of the matrix system for several turbines in interaction may be extended to all these approaches [5,6,10,11,17–19]. Moreover, the presented method is not limited to turbine interactions but may be applied to the computation of several non-deformable moving objects.

In the above mentioned studies, the matrix system basically represents geometrical compounds of the equations to solve. In most cases, when treating one single rigid structure, the matrix is constant over time. In a former study [5], when computing a single rigid turbine, the matrix was a time-constant and matrix inversion was cost effective. A parallel Gauss–Jordan method was then implemented; the matrix inverse was computed once and for all at the beginning of the simulation and stored prior to the unsteady iterations. At each time step, a simple matrix-vector multiplication was used. When computing turbine interactions (starting by two turbines only [6]), as the matrix is no longer a time-constant owing to the relative motion of the two turbines (see Fig. 5), matrix inversion at each time step becomes too expensive in terms of CPU time. Following the literature, we consider iterative methods such as Jacobi, Gauss–Seidel, Conjugate–Gradient (CG) and its variants. In order to choose the best implementation, a matrix characterisation study is performed in Section 3. Important literature exists on CG-class methods with essentially applications to large matrices (up to billions of elements [20]). To have a better understanding of CG methods, the reader may refer to the intuitive and comprehensive lecture note by Shewchuk [21]. The implemented Bi-CGSTAB comes from van der Vorst [22] and the convergence is compared with two other methods, namely an iterative Jacobi method and a classical Conjugate-Gradient. The Bi-CGSTAB appears to be much more efficient with appropriate preconditioning. A suitable and efficient preconditioner, which takes advantage of the block structure of the involved matrix, is derived and appears to considerably accelerate the convergence of the system solve.

The paper is organised as follows. First, the numerical method for flow simulation is presented in Section 2 as in references [5,6]. Then, several matrices are well characterised in Section 3 in order to explain the choice of the Bi-CGSTAB. A specific preconditioner is presented and convergence is analysed in Section 3.2 with and without the proposed preconditioner. Last, in Section 4, the proposed approach is applied to the simulation, in terms of wake characterisation, of elementary interactions between marine current turbines in a farm. Computational times are presented and compared with those obtained using a direct solver (*i.e.* direct matrix inversion).

2. Description of the numerical method

The following paragraphs give the mathematical background regarding the governing equations for the modelling of turbine farms by means of the proposed approach. One can also refer to [5] for further details. The set-up consists of an exterior fluid domain $\mathcal{V}$ with moving boundaries $\mathcal{S}$. Here, the boundaries $\mathcal{S}$ correspond to the surfaces of the turbine blades and hub.